Math 182 Spring 2007 Homework Sheet I.

1. For each function, find all the critical points.

(a) $f(x,y) = x^2 - 6x + 2y^2 - 8y + 5$.

(b) $g(x,y) = x^2 - 4xy - 2y^2 + 2x + 8y + 2$

(c) $f(x,y) = x^2 - 2y^3 + 4xy$

(d) $f(x,y) = x^3 - 3y^2 - 2xy - x$

Ans: (a) (3,2) (b) (1,1) (c) (0,0), (8/3, -4/3)

(d) $((-1+\sqrt{28})/9, (1-\sqrt{28})/27)$, $((-1-\sqrt{28})/9, (1+\sqrt{28})/27)$

2. For each function, find all the critical points. For each critical point, tell whether it is a relative maximum, a relative minimum, or a saddle point.

(a) $f(x,y) = x^2 - 2x + y^2 - 6y + 12$

(b) $f(x,y) = x^2 - 4x - y^2 - 2y + 7$

(c) $f(x,y) = 5 - x^2 - 4x - 2y^2 + 12y + 30$

(d) $f(x,y) = 3xy - 6x$

(e) $f(x,y) = xy - x^2 - y^3$

(f) $f(x,y) = 2xy + 2x^2 - 4y^3$

(g) $f(x,y) = 4xy + x^2 + y^2$

Ans

(a) (1,3) is a relative minimum.

(b) (2, -1) is a saddle point.

(c) (-2, 3) is a relative maximum.

(d) (0,2) is a saddle point.

(e) (0,0) is a saddle point; (1/12, 1/6) is a relative maximum.

(f) (0,0) is a saddle point, (1/24, -1/12) is a relative minimum

(g) (0,0) is a saddle point

3. A flat plate lies on the xy plane. The temperature at (x,y) is

$T(x,y) = x^2 - 6x + y^2 - 10y + 5$ degrees.

Is there a relative minimum temperature or a relative maximum temperature? If so, tell where it occurs, which kind it is, and tell the temperature there.

Ans. There is a relative minimum temperature at (3, 2.5); the temperature there is -22.75 degrees.

4. A flat plate lies on the xy plane. The temperature at (x,y) is

$T(x,y) = 2x^2 - 4x + xy + y^2 - 2y + 5$ degrees.

Is there a relative minimum temperature or a relative maximum temperature? If so, tell where it occurs, which kind it is, and tell the temperature there.

Ans. There is a relative minimum temperature at (6/7, 4/7); the temperature there is 2.714 degrees.

5. A flat plate lies on the xy plane. The concentration of a certain salt at (x,y) is

$C(x,y) = x^2 + 4x + y^2 - 10y + 100$ mg/m^2.

Is there a relative minimum concentration or a relative maximum concentration? If so, tell where it occurs, which kind it is, and tell the concentration there.

Ans. There is a relative minimum concentration at (-2, 5); the concentration there is 71 mg/m^2.

6. A rectangular box with sides x, y, and z (in inches) must satisfy that $x+y+z = 100$. Find the dimensions of the box with the largest volume.

Ans: $x = y = z = 100/3$ inches

7. A rectangular box with sides x, y, and z (in inches) must satisfy that $x+2y+3z = 900$. Find the dimensions of the box with the largest volume.

Ans: $x = 300$ in, $y = 150$ in, $z = 100$ in