1. For each function, find all the critical points.
   (a) \( f(x,y) = x^2 - 6x + 2y^2 - 8y + 5 \).
   (b) \( g(x,y) = x^2 - 4xy - 2y^2 + 2x + 8y + 2 \).
   (c) \( f(x,y) = x^2 - 2y^3 + 4xy \).
   (d) \( f(x,y) = x^3 - 3y^2 - 2xy - x \).
   Ans: (a) (3,2) (b) (1,1) (c) (0,0), (8/3, -4/3) (d) \((-1+\sqrt{28})/9, (1-\sqrt{28})/27\), \((-1-\sqrt{28})/9, (1+\sqrt{28})/27\).

2. For each function, find all the critical points. For each critical point, tell whether it is a relative maximum, a relative minimum, or a saddle point.
   (a) \( f(x,y) = x^2 - 2x + y^2 - 6y + 12 \).
   (b) \( f(x,y) = x^2 - 4x - y^2 - 2y + 7 \).
   (c) \( f(x,y) = 5 - x^2 - 4x - 2y^2 + 12y + 30 \).
   (d) \( f(x,y) = 3xy - 6x \).
   (e) \( f(x,y) = xy - x^2 - y^3 \).
   (f) \( f(x,y) = 2xy + 2x^2 - 4y^3 \).
   (g) \( f(x,y) = 4xy + x^2 + y^2 \).
   Ans
   (a) (1,3) is a relative minimum.
   (b) (2, -1) is a saddle point.
   (c) (-2, 3) is a relative maximum.
   (d) (0,2) is a saddle point.
   (e) (0,0) is a saddle point; (1/12, 1/6) is a relative maximum.
   (f) (0,0) is a saddle point, (1/24, -1/12) is a relative minimum.
   (g) (0,0) is a saddle point.

3. A flat plate lies on the xy plane. The temperature at \((x,y)\) is
   \[ T(x,y) = x^2 - 6x + 2y^2 - 10y + 5 \] degrees.
   Is there a relative minimum temperature or a relative maximum temperature? If so, tell where it occurs, which kind it is, and tell the temperature there.
   Ans. There is a relative minimum temperature at (3, 2.5); the temperature there is -22.75 degrees.

4. A flat plate lies on the xy plane. The temperature at \((x,y)\) is
   \[ T(x,y) = 2x^2 - 4x + xy + y^2 - 2y + 5 \] degrees.
   Is there a relative minimum temperature or a relative maximum temperature? If so, tell where it occurs, which kind it is, and tell the temperature there.
   Ans. There is a relative minimum temperature at (6/7, 4/7); the temperature there is 2.714 degrees.

5. A flat plate lies on the xy plane. The concentration of a certain salt at \((x,y)\) is
   \[ C(x,y) = x^2 + 4x + 2y^2 - 10y + 100 \] mg/m^2.
   Is there a relative minimum concentration or a relative maximum concentration? If so, tell where it occurs, which kind it is, and tell the concentration there.
   Ans. There is a relative minimum concentration at (-2, 5); the concentration there is 71 mg/m^2.

6. A rectangular box with sides \(x, y,\) and \(z\) (in inches) must satisfy that \(x+y+z = 100\). Find the dimensions of the box with the largest volume.
   Ans: \(x = y = z = 100/3\) inches

7. A rectangular box with sides \(x, y,\) and \(z\) (in inches) must satisfy that \(x+2y+3z = 900\). Find the dimensions of the box with the largest volume.
   Ans: \(x = 300\) in, \(y = 150\) in, \(z = 100\) in