17. \[ y'' - 2y' + y = 8e^t \]

\[ y_p = e^t \]

But \( s = ? \)

\[ y'' - 2y' + y = 0 \]

\[ y = e^t \quad r^2 - 2r + 1 = 0 \]

\[ (r - 1)^2 = 0 \quad \text{so } 1 \text{ is a double root} \]

\[ s = 2 \]

1 is a root twice

\[ y_p = e^t \]

\[ y_p' = 2e^t \]

\[ y_p'' = 4e^t \]

\[ 4e^t = 8e^t \]

\[ 2Ae^t = 8e^t \]

\[ A = 4 \]

\[ y_p = 4te^t \]

21. \[ x'' - 4x' + 4x = te^{2t} \]

\[ x_p = e^t (Ae^{2t} + Be^{2t}) \]

\[ x'' - 4x' + 4x = 0, \quad r^2 - 4r + 4 = 0 \]

\[ (r - 2)^2 = 0 \quad 2 \text{ is a double root} \]

\[ s = 2 \]

\[ x_p = t^2 (Ae^{2t} + Be^{2t}) \]

\[ x_p = Ae^{2t} + Bt^2 e^{2t} \]
\[ y''(x) + y(x) = 2^x \]

\[ 2^x = (e^{\ln 2})^x \]

\[ y_p = x A e^{\ln 2} \]

\[ y'' + y = 0 \]

\[ r^2 + 1 = 0 \quad r = \pm i \]

**How many times is \( \ln 2 \) a root?**

\[ s \geq 0 \]

\[ y = A e^{x \ln 2} \]

\[ y' = A e^{x \ln 2} \ln 2 \]

\[ y'' = A (\ln 2)^2 e^{x \ln 2} \]

\[ A (\ln 2)^2 e^{x \ln 2} + A e^{x \ln 2} = e^{x \ln 2} \]

Divide by \( e^{x \ln 2} \)

\[ A (\ln 2)^2 + A = 1 \]

\[ A (\ln 2)^2 + 1 = 1 \]

\[ A = \frac{1}{(\ln 2)^2 + 1} \]

\[ y = \frac{1}{(\ln 2)^2 + 1} e^{x \ln 2} \]

1a]

\[ 4y'' + 11y' + 2y = -2te^{-3t} \]

\[ y_p = t (At + B) e^{-3t} \]

\[ 4r^2 + 11r - 3 = 0 \]

\[ r = \frac{-11 \pm \sqrt{121 - 4(4)(-3)}}{8} \]

\[ r = \frac{-11 \pm \sqrt{169}}{8} \]

\[ r = \frac{-11 \pm 13}{8} \]

\[ r = \frac{-24}{8} = -3 \quad \frac{2}{8} = \frac{1}{4} \]

\[ y'' + y = 0 \]

\[ r^2 + 1 = 0 \quad r = \pm i \]

\[ s \geq 0 \]

\[ y = A e^{x \ln 2} \]

\[ y' = A e^{x \ln 2} \ln 2 \]

\[ y'' = A (\ln 2)^2 e^{x \ln 2} \]

\[ A (\ln 2)^2 e^{x \ln 2} + A e^{x \ln 2} = e^{x \ln 2} \]

Divide by \( e^{x \ln 2} \)

\[ A (\ln 2)^2 + A = 1 \]

\[ A (\ln 2)^2 + 1 = 1 \]

\[ A = \frac{1}{(\ln 2)^2 + 1} \]

\[ y = \frac{1}{(\ln 2)^2 + 1} e^{x \ln 2} \]
How many times is \( e^3 \) a root?

\[ S = 1 \]

\[ y_p = t (Ax + B)e^{-3t} \]

\[ y_p = At^2e^{-3t} + Bte^{-3t} \]

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What about trig?

To find a particular soln of

\[ ay'' + by' + cy = Ct^n e^{xt} \cos \beta t \]

or \( Cte^{xt} \sin \beta t \)

The guess is

\[ y_p(t) = t^s \left( \sum_{m=0}^{n} A_m t^m + \sum_{n=1}^{m} A_0 e^{xt} \right) e^{xt} \cos \beta t \]

\[ + t^s \left( \sum_{m=1}^{n} B_m t^m + \sum_{n=1}^{m} B_0 e^{xt} \right) e^{xt} \sin \beta t \]

Where \( S \) is the number of times \( x + i\beta \) is a root of the auxiliary eqn.

\[ r = x + i\beta \]

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**Ex:** Find a particular soln to

\[ y'' + 2y' + 2y = 3 \cos t \]

\( x = 0 \)

\[ y_p = t^s \left( A \cos t + B \sin t \right) \]

\[ B = 1 \]

How many times is \( i \)

A root of

\[ r^2 + 2r + 2 = 0 \]

\[ r = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i \]

\( i \) is not a root, \( S = 0 \)

\[ y_p = A \cos t + B \sin t \]
\[ y_p' = -A \cos t + B \cos t \]
\[ y_p'' = -A \cos t - B \cos t \]
\[ -A \cos t - B \sin t - 2A \sin t + 2B \cos t \]
\[ + 2A \cos t + 2B \sin t = 3 \cos t \]
\[ (A + 2B + 2A)(\cos t) + (-A - 2A + 2B) \sin t = 3 \cos t \]

Coeff of \( \cos t \) is \( A + 2B = 3 \)
Coeff of \( \sin t \) is \( B - 2A = 0 \)

\[ B = 2A \]
\[ A + 2(2A) = 3 \]
\[ 5A = 3 \]
\[ A = \frac{3}{5} \]
\[ B = 2A = 2 \left( \frac{3}{5} \right) = \frac{6}{5} \]

\[ y_p(t) = \frac{3}{5} \cos t + \frac{6}{5} \sin t \]

\( \text{Ep Find Form of the Guess for a Particular Solution} \)

\[ y'' + y = 2t^2 \cos t \]
\[ y_p = t^3 (A + 2B + 6t + C) \cos t \]
\[ + t^3 (D + 2B + E + F) \sin t \]

Root is \( \pm \)
\( s = \text{How Many Times is} \ \pm \ \text{a Root of} \)
\[ y'' + y = 0 \]
\[ r^2 + 1 = 0 \]
\[ r^2 = -1, \ r = \pm \iota \]
\( s = 1 \)

Guess \( y_p = \frac{1}{t} \]
\( A + 3 \cos t + Bt^2 \cos t + Ct \cos t \]
\[ + Dt^3 \sin t + Et^2 \sin t + Ft \sin t \]

\( t^2 \cos t \): Remove \( t^2 \): \( \cos t \)
\[ r = x + (\iota B) \]
\[ \cos \beta t \]
BY FIND FORM

\[ y'' + y = 2te^{3t} \cos(4t) \]
\[ y_p = t^{3}(Ae^{3t}\cos(4t) + Be^{3t}\cos(4t)) \]
\[ Cte^{3t}\sin(4t) + De^{3t}\sin(4t) \]

To get \( e^{3t}\cos(4t) \) we root
\[ \lambda_{1,2} = 3 \pm 4i \]

**S2 HOW MANY TIMES** \( 3+4i \) IS A ROOT OF \( r^2 + 1 = 0 \)
\[ r = 0, -i \]
\[ y = Ae^{3t}\cos(4t) + Be^{3t}\cos(4t) + Cte^{3t}\sin(4t) + De^{3t}\sin(4t) \]

Notice I can't do by this method.
\[ y'' + y = \tan t \]

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**THEOREM** suppose \( y_1 \) and \( y_2 \) are linearly independent solutions of
\[ ay'' + by' + cy = 0 \]
AND \( y_p(t) \) is a particular solution of
\[ ay'' + by' + cy = f(t) \]
THEN THE GENERAL SOLUTION OF
\[ ay'' + by' + cy = f(t) \]
IS
\[ y = c_1y_1(t) + c_2y_2(t) + y_p(t) \]

Just add in soln of homogeneous eqn.
\[4y'' + 11y + 23y = -2 + e^{-3t}\]

General solution:
\[y_p = \frac{1}{5} \cos t + \frac{6}{5} \sin t\]

But homogenous:
\[4y'' + 11y' + 3y = 0\]

17) \[y'' - 2y' + y = 8e^t\]

From general soln:
\[y_p = 4t^2 e^t\]
\[y'' - 2y' + y = 0\]
\[r^2 - 2r + 1 = 0\]
\[(r - 1)^2 = 0\]
\[r = 1, 1\]

Homogeneous soln:
\[y_1 = e^t\]
\[y_2 = C_1 e^t + C_2 t e^t\]

Answer:
\[y = C_1 e^t + C_2 t e^t + 4t^2 e^t\]