\[ \frac{dx}{dt} = xy \]
\[ \frac{dy}{dt} = x^2 + y^2 - 1 \]

Critical Points

\[ x' = 0 \]
\[ y' = 0 \]
\[ x - y = 0 \]
\[ x^2 + y^2 - 1 = 0 \]
\[ x = y \]

\[ y^2 + y^2 - 1 = 0 \]
\[ 2y^2 = 1 \]
\[ y^2 = \frac{1}{2} \]
\[ y = \pm \sqrt{\frac{1}{2}} \]

1. \[ x = y = \sqrt{\frac{1}{2}} \]
2. \[ x = y = -\sqrt{\frac{1}{2}} \]

\[ \left( \sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}} \right) \]
\[ \left( -\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}} \right) \]

\[ \frac{dy}{dt} = 2y \]
\[ \frac{dx}{dt} = 2x \]

\[ \frac{dy}{dx} = \frac{y}{x} \]

\[ y \, dx = x \, dy \]
\[ y^2 = x^2 + C \]

\[ y^2 - x^2 = k \]
\[ y^2 - x^2 = 1 \]
\[ y^2 - x^2 = -1 \]
\[ x^2 - y^2 = 1 \]

Hyperbolas
\[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \]
\[ \frac{dy}{dt} = y - 1 \]
\[ \frac{dy}{dx} = e^{x+y} \]
\[ dy = \frac{e^{x+y}}{y-1} \cdot dx \]
\[ (y-1)dy = e^{x+y} \cdot dx \]

\[ \text{PART} \]
\[ \int e^{-y} (y-1) \, dy = \int e^x \, dx \]
\[ \int e^{-y} \, y \, dy - \int e^{-y} \, dy = e^x + C \]
\[ u = y, \quad dv = e^{-y} \, dy \]
\[ du = dy, \quad v = -e^{-y} \]
\[ y(-e^{-y}) - \int -e^{-y} \, dy = \int e^{-y} \, dy = e^x + C \]
\[ -ye^{-y} = e^x + C \]

Sometimes higher order equations are solved using systems

\[ y'' + y^3 = 0 \]

We introduce a new variable \( V \)

\[ V = y' \]

Then \( V' = y'' = -y^3 \)

So this is really the system

\[ y' = V \]
\[ V' = -y^3 \]
NAMES FOR KINDS OF "STABILITY"

A critical point \((x, y)\) is **asymptotically stable** if the solution curves that start near \((x_0, y_0)\) converge to \((x_0, y_0)\).

It is **unstable** if there are nearby points such that trajectories starting there tend to go far away from the critical point.

It is **stable** if nearby points tend to stay close but don't converge to the critical point.
Names for types of critical points

1. Center
   \((x_0, y_0)\)
   Trajectories near \((x_0, y_0)\) stay on closed curves.

2. Node
   All trajectories near \((x_0, y_0)\) tend to \((x_0, y_0)\) in definite directions, or tend away from \((x_0, y_0)\) in definite directions.

Asymptotically stable node

Unstable node

Asymptotically stable node
3. **Spiral Point**: Trajectories are spirals. Asymptotically stable spiral.

4. **Saddle Point**: There is a direction where trajectories move towards critical point. There is a direction so trajectories move out. Unstable.