Nonlocal School on Fractional Equations
NSFE 2017
Abstracts

Department of Mathematics
Iowa State University

August 17–19, 2017
Numerical methods for fractional diffusion

Ricardo H. Nochetto
University of Maryland, College Park

We present and analyze three finite element methods (FEMs) for the numerical approximation of fractional diffusion in bounded domains in any dimensions. The first FEM deals with the fractional spectral Laplacian and hinges on the extension to an infinite cylinder in one more dimension. The second FEM concerns the integral formulation of fractional Laplacian in the entire space. The third FEM is a Dunford-Taylor approach which applies to both definitions. We discuss rather delicate numerical issues that arise in the construction of reliable FEMs and in the a priori and a posteriori error analyses of such FEMs for both steady and evolution fractional diffusion, show illustrative simulations, and mention challenging open questions.
Conferences

An optimization-based coupling strategy for classical and nonlocal elasticity

Marta D’Elia
Sandia National Laboratories

The use of nonlocal models in science and engineering applications has been steadily increasing over the past decade. The ability of nonlocal theories to accurately capture effects that are difficult or impossible to represent by Partial Differential Equations (PDEs) motivates and drives the interest in this type of simulations. However, the improved accuracy of nonlocal models comes at the price of a significant increase in computational costs. As a result, it is important to develop local-to-nonlocal coupling strategies, which aim to combine the accuracy of nonlocal models with the computational efficiency of PDEs. We develop an optimization-based method for the coupling of nonlocal and local problems in the context of nonlocal elasticity. The approach formulates the coupling as a control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the nonlocal and local domains, and the controls are virtual volume constraints and boundary conditions. Numerical results for nonlocal diffusion and nonlocal elasticity in three-dimensions illustrate key properties of the optimization-based coupling method; these numerical tests provide the groundwork for the development of efficient and effective engineering analysis tools.
Hölder and Schauder estimates. Pointwise and semigroup strategies.

Marta de León-Contreras
Universidad Autónoma de Madrid

In this talk we shall give Hölder and Schauder estimates for discrete fractional derivatives as well as for the fractional parabolic harmonic oscillator.

We present the discrete fractional derivatives and integrals, and we show some regularity results when the space is a mesh of length \( h \). In this case, see [1], Hölder and Schauder estimates are obtained by means of pointwise estimates. This kind of results have been obtained recently for the fractional discrete Laplacian, see [2].

For the parabolic harmonic oscillator, the estimates are obtained by using semigroup theory. In fact the “adapted spaces” to this operator are defined by

\[
\Lambda_{t,H_x}^{\alpha/2,\alpha} := \{ f \in L^\infty(\mathbb{R}^{n+1}) : \left\| \frac{\partial^k}{\partial y^k} P_y f \right\|_{L^\infty(\mathbb{R}^{n+1})} \leq C y^{-k+\alpha}, \ y > 0 \}, \ \alpha > 0.
\]

Here \( k = \lfloor \alpha \rfloor + 1 \) and \( P_y \) is the Poisson semigroup associated with the operator. When the operator is the Laplacian, the parabolic “adapted spaces” \( \Lambda_{t,x}^{\delta/2,\delta} \) are defined in [4], where it is shown that they coincide with the parabolic spaces \( C^{\delta/2,\delta} \) introduced by Krylov in [3], when \( \delta \) is not an integer. In our case, when \( \alpha \) is not an integer, the spaces \( \Lambda_{t,H_x}^{\alpha/2,\alpha} \) coincide with a version of parabolic spaces \( C^{\delta/2,\delta} \) defined as in Krylov, but adapted to the harmonic oscillator. In addition, if the function \( f \) does not depend on \( t \), the spaces coincide with the Hermite Hölder spaces defined in [5].

Finally we shall show an approximation theorem of the discrete fractional derivatives to the continuous fractional derivatives, for functions in the discrete Hölder spaces. This result also allows us to prove the coincidence, for good enough functions, of the Marchaud and Grünwald-Letnikov fractional derivatives at every point and the speed of convergence to the Grünwald-Letnikov fractional derivative, see [1].

References


Optimization with respect to order in a fractional diffusion model: analysis and approximation

Abner J. Salgado
The University of Tennessee, Knoxville

We consider an identification problem, where the state $u$ is governed by a fractional elliptic equation and the unknown variable corresponds to the order $s \in (0,1)$ of the operator. We study the existence of an optimal pair $(\bar{u}, \bar{s})$ and provide sufficient conditions for its uniqueness. We develop semi-discrete and fully discrete algorithms to approximate the solution and provide an analysis of their convergence properties. We present numerical illustrations that confirm and extend our theory. This is joint work with E. Otárola and H. Antil.

Nonlocal mechanics models for anisotropic media

Pablo Seleson
Oak Ridge National Laboratory

Peridynamics is a nonlocal reformulation of classical continuum mechanics, suitable for material failure and damage simulation. Originally, this nonlocal theory was presented as the bond-based peridynamics theory, for which the material response of an isotropic medium is limited by a fixed Poisson’s ratio. To overcome this limitation, the state-based peridynamics theory was developed. Applications in peridynamics to date cover a wide range of engineering problems; however, the majority of those applications employ isotropic material models. Only recently, a limited number of anisotropic peridynamic models were developed. In this talk, we will first survey the different classes of anisotropic material models in classical linear elasticity, and we will present a peridynamic framework to represent anisotropic materials. We will then show a classification and a hierarchy of anisotropic peridynamic models, and we will discuss their relation to classical elasticity as well as restrictions arising from a bond-based interaction assumption.
Regularity theory for non local in time operators
Alexis F. Vasseur
The University of Texas at Austin

In this talk, we will present new applications of the De Giorgi method to show the regularity of solutions to nonlocal operators. We will focus on the case of fractional derivatives in time. Those equations are important for the modeling of memory effects as hysteresis. This is a joint work with Mark Allen and Luis Caffarelli.

What are the classical boundary conditions for the fractional Laplace operator?
Mahamadi Warma
University of Puerto Rico (Rio Piedras Campus)

In this talk we characterize all the classical boundary conditions (Dirichlet, Neumann and Robin) associated with the fractional Laplace operator or/and the regional fractional Laplace operator on bounded subsets of $\mathbb{R}^N$. We also give some well-posedness and regularity results of solutions to the associated elliptic and parabolic problems. Finally we introduce a fractional Dirichlet to Neumann operator associated with the regional fractional Laplacian.
## NSFE 2017 Preliminary Schedule

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