Homework 7 solutions

See also Excel file HW709sol.xls

1) \[ P = \begin{bmatrix} 0 & .25 & .75 \\ .5 & 0 & .5 \\ .3 & .7 & 0 \end{bmatrix} \]

\[ v_0 = .5v_1 + .3v_2 \quad v_0 = 1 \]
\[ v_1 = .25v_0 + .7v_2 \quad v_1 = 1.192 \]
\[ v_2 = 1.346 \]

(a) From convolution algorithm: \( TH(3) = 0.751 = \frac{\hat{q}(2)}{\hat{q}(3)} \) (Excel)

(b) From MVA \( TH(3) = 0.751 \)

8.11) For the two part types aggregated into 1, the transition matrix is

0 1 2 3
0 \[ \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \] Since loading order maintains 1:2 ratio use \( D_1:D_2 = 1:2 \)
1 \[ \begin{bmatrix} 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \] in place of \( \lambda(\ell) \) in section 7.3.2 aggregation procedure
2 \[ \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \]
3 \[ \begin{bmatrix} \frac{3}{5} & \frac{2}{5} & 0 & 0 \end{bmatrix} \]

\[ \lambda_{01} = 1, \quad \lambda_{02} = 2 \]
\[ \lambda_{12} = 1, \quad \lambda_{13} = 2 \]
\[ \lambda_{23} = 1 + 2, \quad \lambda_2 = 1 + 2 \]
\[ \lambda_{30} = 3, \quad \lambda_{31} = 2 \]
\[ \lambda_3 = 1 + 2(2) = 5, \quad p_{23} = 1 \]

\[ v_0 = 1 \]
\[ v_1 = \frac{1}{3} + \frac{2}{3} v_3 \quad v_0 = 1 \]
\[ v_2 = \frac{2}{3} + \frac{1}{3} v_1 \quad v_2 = 1 \]
\[ v_3 = \frac{2}{3} v_1 + v_2 \quad v_3 = \frac{5}{3} \]

Solve using mean value analysis for \( n = 6 \) parts (see Excel file)

\( TH_1(6) = \frac{1}{7} TH(6) = 0.188 \)
\( TH_2(6) = \frac{2}{7} TH(6) = 0.377 \)
8.12) Now we have

\[
P^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \end{bmatrix} \quad \quad P^{(2)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}
\]

\[v_0^{(1)} = v_1^{(1)} = v_2^{(1)} = v_3^{(1)} = 1 \quad \quad v_0^{(2)} = v_1^{(2)} = v_2^{(2)} = 1, \quad v_3^{(2)} = 2\]

See Excel file for multi-chain MVA

TH1(4,2) = .250, TH2(4,2) = .331

5) Since there are 4 kanbans circulating in 2 machines, there are five possible states.

\[S_5 = \{(0,4), (1,3), (2,2), (3,1), (4,0)\}\] and can be modeled as CTMC.

\[
\begin{align*}
2.5p(4,0) &= 1p(3,1) \\
(1 + 2.5)p(3,1) &= 2.5p(4,0) + 2p(2,2) \\
(2 + 2.5)p(2,2) &= 2.5p(3,1) + 2p(1,3) \\
(2 + 2.5)p(1,3) &= 2.5p(2,2) + 2p(0,4) \\
2p(0,4) &= 2.5p(1,3) \\
p(0,4) + p(1,3) + p(2,2) + p(3,1) + p(4,0) &= 1
\end{align*}
\]

Solve the balance equation to find the probability distribution of the number of jobs at each stage.

\[p(4,0) = 0.064876, p(3,1) = 0.16219, p(2,2) = 0.202737, p(1,3) = 0.253421, p(0,4) = 0.316776\]

a) \[\rho_1 = 1 - p(0,4) = .683224\]

\[\rho_2 = 1 - p(4,0) - .5p(3,1) = .854\]

b) \[P(N_2(t) \geq 3) = p(0,4) + p(1,3) = .570198\]