

# A GENERAL QUEUING NETWORK MODEL TO OPTIMIZE REFURBISHING POLICIES

Jumpol Vorasayan, Nanyang Technological University  
Sarah M. Ryan, Iowa State University

## Abstract

A generalized queueing model simulates a closed-loop supply chain that includes manufacturing, evaluation by consumers, return, refurbishment and resale. We examine the optimal supply curves for refurbished products resold back to consumers along with the corresponding total profit with respect to four attributes: 1) variability of manufacturing and refurbishing times, 2) the number of times a product may be refurbished and resold, 3) product return probabilities, and 4) prioritizing returns for refurbishment by quality grade. The optimal supply curves for refurbished products and the total profit are parameterized by consumer willingness to pay for refurbished products. Variability in manufacturing times encourages expanded supply of refurbished products, while variability in refurbishing times has the opposite effect. In some cases, it is optimal to allow refurbishing at most once. The proportion of previously refurbished products that are returned (i.e., accepted) again has a pronounced negative impact on profit and causes an optimally smaller number of refurbished products to be offered at a higher price. Savings from prioritizing returns for refurbishment are significant when it is optimal to refurbish only a portion of returns.

# A GENERAL QUEUING NETWORK MODEL TO OPTIMIZE REFURBISHING POLICIES

*Jumpol Vorasayan*

*Center for Supply Chain Management, Nanyang Technological University, Singapore*

*and*

*Sarah M. Ryan*

*Industrial & Manufacturing Systems Engineering, Iowa State University, Ames, IA 50011-2164*

## 1. Introduction

A significant portion, estimated at 5 to 35 percent, of products are returned to the original equipment manufacturer (OEM) shortly after sales [1]. Reasons for these commercial returns include customer dissatisfaction or remorse, defects covered by warranties, or end of lease [2]. The value of these returns is considerable due to substantial useful life remaining since most of these returns have been used only lightly and still preserve most features of brand new products. A reverse supply chain may be created to redeem the cost or make as much profit as possible from returned products. The forward distribution combined with reverse collection forms the closed-loop supply chain which has received increasing attention recently.

The model in this paper is developed for OEMs who assemble products to order and refurbish products to [3], [4]. These OEMs include automotive and computer manufacturers. For example, computer manufacturers, such as Dell and Gateway, assemble new desktop and laptop computers when specific orders arrive to satisfy consumers' individual preferences. Returned products, on the other hand, are either dismantled or refurbished and placed in a warehouse, ready to ship immediately when demand arrives.

Of the several recovery process options, refurbishment is the one that specifically brings commercial returns back to acceptable condition for resale [3]. Consumers typically are not

willing to pay as much for refurbished products as for new ones. Although the refurbished products are able to function as well as new, their pre-owned status diminishes their perceived quality. Introducing refurbished products to the market might increase revenue to OEMs more than dismantling and selling them in parts. However, it can also cannibalize the demand for new products because consumers have the choice to buy either new or refurbished products (see previous market segmentation papers by Debo et al. [6], Ferguson and Toktay [7] and Vorasayan and Ryan [8]). The price of refurbished products should be placed low enough to draw potential consumers but not lower than the point where most consumers in the market would buy refurbished instead of new products. On the other hand, the price must be set sufficiently high so that refurbishing is a profitable strategy for OEMs. While the demand of refurbished products can be controlled by price, the supply is decided by the proportion of returns to be refurbished. These two variables should be decided upon together with care to avoid excess inventories and customer dissatisfaction due to deficient supply. Ferrer and Swaminathan [9] explore the optimality conditions for new/remanufactured product price and their quantity for original equipment manufacturer and independent operator. Bayindir et al. [10] propose a single period model to analyze the profitability condition for tire industry under limited production capacity and constant proportion of substitutable demand.

Matching supply and demand is only one of several aspects in reverse supply chain design [11]. The other aspects include the high level of uncertainty in timing, quantity and quality of returned products along with their highly variable processing times. Several types of queueing models have been applied in the remanufacturing environment to address the uncertainty issues. The advantages of queueing models include: incorporating correlation between sales and returns; and permitting the accounting of costs associated with the

refurbishing policy throughout the system including transportation, handling and holding inventories of rapidly depreciating products. Guide and Gupta [12] developed queueing models to estimate the flow time of remanufacturing products flowing through three different segments: disassembly, remanufacturing and reassembly. The model yielded results fairly similar to those from simulation. Toktay et al. [13] constructed a queueing network to simulate the entire supply chain of a single-use camera. The proposed heuristic found the optimal ordering policy to minimize the costs of procurement, inventory, and lost sales in cases of trackable and untrackable time of purchase. Bayindir et al. [14] found the optimal probability of return for items sold. They assumed that the returns are controllable and the OEM has infinite capacity. Unlike our model, the consumers are indifferent between the new and remanufactured products. Souza et al.[15] simulated the remanufacturing facility as a  $GI/G/1$  queueing network. The product returns had different grades that require different processing times. The model found the optimal product mix for remanufacturing to maximize profit. Ketzenberg and Souza [16] compared two configurations of a remanufacturing process, mixed and parallel lines, in several scenarios by using a  $GI/G/m$  queueing network.[2] analyzed and suggested the appropriate supply chain for commercial product returns of products with different rates of decay in price. Lieckens and Vandaele [17] included inventory holding cost in a facility allocation problem in the reverse logistics environment by combining a  $GI/G/m$  queueing model with a traditional mixed integer linear programming model.

In this paper, we propose a generalized queueing network (GQN) model that differs from previous work by addressing both market segmentation and uncertainty in manufacturing at the same time without restrictive assumptions on random variable distributions. We experiment with the variability in the durations of manufacturing, refurbishing, storage and evaluation processes.

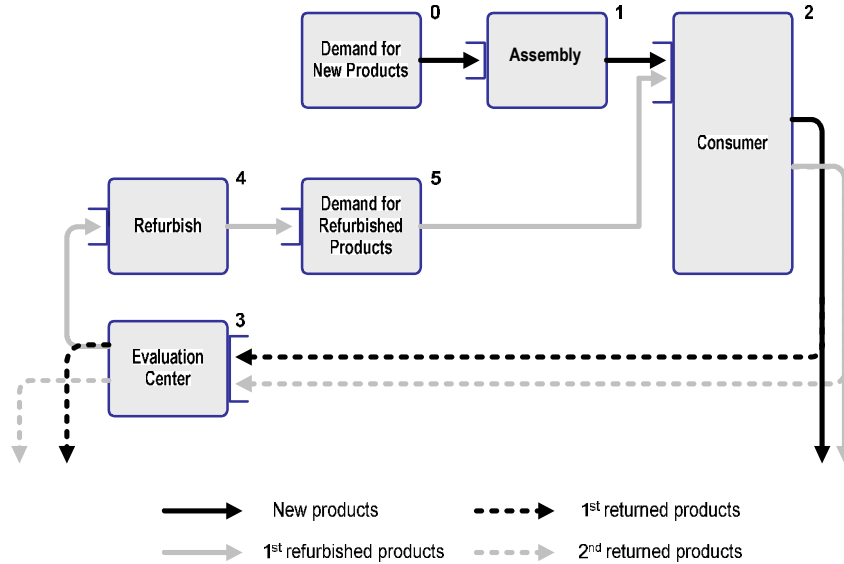
Moreover, the model allows any restriction on the number of times an item can be refurbished rather than limited to once [6] or unlimited ([13], [2], [8]). For example, products such as disposable cameras can be remanufactured only a certain number of times [18]. The return probability and quality distribution can be assigned differently depending on the number of times a product has been returned previously. These advantages allow us to study new aspects of the closed-loop supply chain system. As a result, we can study the effects on total profit and the supply curve of refurbished products of (1) the variability in the times during which products are manufactured, refurbished, stored in inventory and used by consumers, (2) the returned products' refurbishing history, (3) the probabilities of products being returned, and (4) sorting and prioritizing returns for refurbishment.

The remainder of this research is organized as follows: Section 2 provides the mathematical model and Section 3 shows numerical examples that illustrate the implementation of the model and its results. Section 4 concludes with a description of future work.

## **2. Model Formulations and Optimality Conditions**

The lowest tier of the supply chain including refurbishing is formulated as a queueing network (Figure 1). New products are assembled according to orders at station 1, then distributed to consumers which are represented by station 2. Some products are returned within their warranty period from consumers to station 3, the evaluation center where the decision is made whether to refurbish or dismantle the products. Products are refurbished at station 4 and kept for sale at station 5. We use the word “job” as a general term to represent an entity flowing in the queueing network. Jobs at station 1 are orders waiting to be processed while jobs at stations 2, 3, 4 and, 5 are finished products which can be new, returned or refurbished products.

Jobs are categorized into different types as described in Table 1 according to their routings and the number of times they have been returned. Let  $\eta$  be the maximum number of times that a product may be refurbished and resold to consumers. When  $\eta = 0$ , the system is the forward supply chain that consists only of stations 1 and 2. When a job type  $\eta'$  is returned, it becomes a type  $\eta + 1$  and is automatically sent to be dismantled. Additionally, jobs of each type transferring from station 3 to 4 will be evaluated and further classified into  $\theta$  grades, similarly to [4], [15]. We denote jobs of type  $l$  and grade  $r$  as jobs of class  $(l, r)$ .



**Figure 1. The open queueing network for  $\eta = 1$ .**

Jobs arriving at station 1, 3, 4, and 5 are assumed to be served one-by-one by a single server. On the other hand, station 2 is modeled with infinite servers because the make-to-order policy guarantees that every new product produced will be distributed and evaluated immediately by the consumers who order them. The model is a multiple-class general open queueing network. We base our performance evaluation on the parametric-decomposition method for multiple-class

$G/G/m_i$  open queueing network developed similarly in [19], [20], [21], and [22]. The approximation technique yields comparable results with those from simulation [21] [23].

**TABLE 1. Job types and routings.**

Job type	Description	Routing
0	New products that are kept by the consumer after purchase.	{0,1,2,0}
$l$	Products that have been returned for the $l^{\text{th}}$ time from consumers and are sent to be dismantled	{2,3,0}
$l'$	Returned products that have been refurbished for the $l^{\text{th}}$ time before being dismantled	{3,4,5,2,0}

The parametric-decomposition method evaluates each station  $i$  individually as a stochastically independent  $GI/G/m_i$  system by the set of parameters  $\{\lambda_i, c_{a_i}^2, E[S_i], c_{s_i}^2\}$ , where  $\lambda_i =$  expected arrival rate at station  $i$ ,  $c_{a_i}^2 =$  squared coefficient of variation (scv) of the external interarrival time at station  $i$ ,  $E[S_i] =$  expected service time of aggregate jobs at station  $i$ ,  $c_{s_i}^2 =$  scv of the service time of aggregate jobs at station  $i$ . A summary of the notation can be found in Appendix A.

For job types  $l, k = 0, 1, 1', \dots, \eta, \eta', \eta + 1$  and stations  $i, j = 1, \dots, 5$ , let  $\lambda_i^{(l)}$  = the expected arrival rate of job type  $l$  to station  $i$ ,  $\lambda_{0i}^{(l)}$  = the external arrival rate of job type  $l$  to station  $i$ ,  $p_{ij}^{(k)(l)}$  = the probability that a job leaving station  $i$  as a type  $k$  job will join station  $j$  as a type  $l$  job, and  $p_{ij}$  = the transfer probability of aggregate jobs from station  $i$  to station  $j$ . Let  $q^{(l,r)}$  = the proportion of  $l^{\text{th}}$ -return products with grade  $r$ , such that  $\sum_{r=1}^{\theta} q^{(l,r)} = 1$  for all  $l$ . From station 3 to station 4, let  $p_{mr}^{(l,r)}$  = the proportion of class  $(l,r)$  products that OEM decides to refurbish,  $p_{mr}^{(\bullet)}$  = the proportion of  $l^{\text{th}}$ -return products (any grade) to be refurbished, and  $p_{mr}^{(\bullet,r)}$  = the proportion of

grade  $r$  (any type) to be refurbished. These proportions are decision variables and are treated as routing probabilities in the queuing network.

Let  $p_{cr}^{(l\bullet)}$  the probability that a job type  $l$  (aggregated over grades) will be returned from the consumer to the manufacturer. In the queuing network, for  $l=1,\dots,\eta$ , the other routing probabilities are  $p_{20}^{(l'-1)(l'-1)} = 1 - p_{cr}^{(l\bullet)}$ ,  $p_{23}^{(l'-1)(l)} = p_{cr}^{(l\bullet)}$ ,  $p_{30}^{(l)(l)} = 1 - p_{mr}^{(l\bullet)}$ ,  $p_{34}^{(l)(l')} = p_{mr}^{(l\bullet)} = \sum_{r=1}^{\theta} q^{(l,r)} p_{mr}^{(l,r)}$ . Also,  $p_{01}^{(0)(0)} = p_{12}^{(0)(0)} = 1$ ,  $p_{20}^{(0)(0)} = 1 - p_{cr}$ ,  $p_{23}^{(0)(1)} = p_{cr}^{(l\bullet)}$  and  $p_{34}^{(0)(0')} = p_{mr}^{(0)} = 1$ .

Because all products that have been returned  $\eta + 1$  times are automatically dismantled,  $p_{30}^{(\eta+1)(\eta+1)} = 1$ . The only external arrivals to the system occur at station 1, where demands (orders) for new products arrive, so that  $\lambda_{01}^{(0)} = \lambda_{new}$ . The demands for new and refurbished products, respectively, are given by  $\lambda_{new} = 1 - \max\left(P_{new}, \frac{P_{new} - P_{ref}}{1 - \delta}\right)$  and  $\lambda_{ref} = \max\left(\frac{P_{new} - P_{ref}}{1 - \delta} - \frac{P_{ref}}{\delta}, 0\right)$  where  $P_{new}$  is the exogenous price of new products,  $P_{ref}$  is the decision variable price of refurbished products, and  $\delta$  is the consumer willingness-to-pay of refurbished products,  $0 < \delta < 1$ . More details of demand functions can be found in [8].

From the traffic rate equations in steady state,  $\lambda_i^{(l)} = \lambda_{0i}^{(l)} + \sum_{j=1}^5 \sum_k \lambda_j^{(k)} p_{ji}^{(k)(l)}$ ,  $i = 1, \dots, 5$ , we obtain  $\lambda_1^{(0)} = \lambda_2^{(0)} = \lambda_{new}$ ,  $\lambda_2^{(l')} = \lambda_4^{(l')} = \lambda_5^{(l')} = \lambda_{new} \prod_{k=1}^l p_{cr}^{(k\bullet)} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)})$ ,  $l = 1, \dots, \eta$ , and  $\lambda_3^{(l)} = \lambda_{new} \prod_{k=1}^l p_{cr}^{(k\bullet)} \sum_{r=1}^{\theta} (p_{mr}^{(k-1,r)} q^{(k-1,r)})$ ,  $l = 1, \dots, \eta + 1$ . The arrival rates of aggregate jobs to station  $i$ ,  $\lambda_i = \sum_l \lambda_i^{(l)}$ , are  $\lambda_1 = \lambda_{new}$ ,  $\lambda_2 = \lambda_{new} \left(1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k\bullet)} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)})\right)$ ,  $\lambda_3 =$

$\lambda_{new} p_{cr} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k+1, \bullet)} \sum_{r=1}^{\theta} \left( p_{mr}^{(k,r)} q^{(k,r)} \right) \right)$  and  $\lambda_4 = \lambda_5 = \lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k, \bullet)} \sum_{r=1}^{\theta} \left( p_{mr}^{(k,r)} q^{(k,r)} \right) \right)$ . The

transition probability matrix of aggregate jobs can be obtained by  $p_{ij} = \frac{\lambda_{ij}}{\lambda_i}$ , where

$\lambda_{ij}^{(l)} = \sum_k \lambda_i^{(k)} p_{ij}^{(k)(l)}$  and  $\lambda_{ij} = \sum_l \lambda_{ij}^{(l)}$ ,  $i, j = 0, \dots, 5$ ,  $k, l = 0, 1, 1', \dots, \eta, \eta', \eta + 1$ . Note that when  $\eta < \infty$ ,

$$p_{34} = \frac{\lambda_{34}}{\lambda_3} = \frac{\lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k, \bullet)} \sum_{r=1}^{\theta} \left( p_{mr}^{(k,r)} q^{(k,r)} \right) \right)}{\lambda_{new} p_{cr}^{(1)} \left( 1 + \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k+1, \bullet)} \sum_{r=1}^{\theta} \left( p_{mr}^{(k,r)} q^{(k,r)} \right) \right)} < 1 \text{ because product returned for the}$$

$(\eta + 1)$ th time can no longer be refurbished and will be automatically sent to be dismantled.

The parameters  $E[S_i]$  and  $c_{s_i}^2$  for  $i = 1, 2, 3$  represent the expected value and variance of time for manufacturing new products, consumer evaluation before return, if any, and duration for which returned products are kept in inspection centers respectively. Let  $\rho_i$  be the expected utilization of station  $i$ , where  $\rho_i = \lambda_i E[S_i]$ . The expected service time at station 4 is a weighted

average according to the arrival rate of each grade:  $E[S_4] = \frac{1}{\lambda_4} \sum_{r=1}^{\theta} \lambda_4^{(\bullet r)} E[S_4^{(\bullet r)}]$ , where  $E[S_4^{(\bullet r)}]$

is the expected service time of grade  $r$ , and

$$E[S_4^{(\bullet r)}] = \begin{cases} \frac{1}{\lambda_4^{(\bullet r)}} \sum_{l=1}^{\eta} \lambda_4^{(l,r)} E[S_4^{(l,r)}] & \text{if } \sum_l p_{mr}^{(l,r)} > 0 \\ 0 & \text{otherwise} \end{cases}.$$

The aggregate arrival rate for grade  $r$ ,  $\lambda_4^{(\bullet r)} = \lambda_{new} \left( \sum_{l=1}^{\eta} p_{mr}^{(l,r)} q^{(l,r)} \prod_{k=1}^l p_{cr}^{(k, \bullet)} \sum_{r=1}^{\theta} \left( p_{mr}^{(k-1,r)} q^{(k-1,r)} \right) \right)$ . The

scv of the refurbishing time is:

$$c_{s_4}^2 = \begin{cases} \left( \frac{\sum_{r=1}^{\theta} \frac{\lambda_4^{(r)}}{\sum_{r=1}^{\theta} \lambda_4^{(r)}} \left( \frac{E[S_4^{(r)}]}{E[S_4]} \right)^2 \left( 1 + \sum_{l=1}^n \frac{\lambda_4^{(l,r)}}{\lambda_4^{(r)}} \left( \frac{E[S_4^{(l,r)}]}{E[S_4^{(r)}]} \right)^2 (1 + c_{s_4}^{2(l,r)}) - 1 \right)}{\sum_{r=1}^{\theta} \frac{\lambda_4^{(r)}}{\sum_{r=1}^{\theta} \lambda_4^{(r)}} \left( \frac{E[S_4^{(r)}]}{E[S_4]} \right)^2} - 1 & \text{if } \sum_l p_{mr}^{(l,r)} > 0 \\ \sum_{r=1}^{\theta} \frac{\lambda_4^{(r)}}{\sum_{r=1}^{\theta} \lambda_4^{(r)}} \left( \frac{E[S_4^{(r)}]}{E[S_4]} \right)^2 - 1 & \text{otherwise} \end{cases}$$

At station 5,  $E[S_5]$  is the average time between demands for refurbished products or its residual in case of a product arrival to an empty store (when the potential sale is lost), i.e.,  $E[S_5] = \frac{1}{\lambda_{ref}}$ . The proportion of lost sales can be computed as the proportion of time that station 5 is idle,  $1 - \rho_5$ . The service time distribution at this station is assume to be exponentially distributed; therefore,  $c_{s_5}^2 = 1$ . The process to obtain parameters  $c_{a_i}^2$  can be found in Appendix B.

The expected waiting time of aggregate jobs in each station can be approximated by

$$E[W_i]_{GI/G/m_i:FCFS} \approx \frac{(c_{a_i}^2 + c_{s_i}^2)g_i}{2} E[W_i]_{M/M/m_i:FCFS}, \quad i = 1, \dots, 5, \quad \text{where } g_i =$$

$$\begin{cases} \exp\left[-\frac{2(1-\rho_i)(1-c_{a_i}^2)^2}{3\rho_i(c_{a_i}^2 + c_{s_i}^2)}\right], & c_{a_i}^2 < 1 \\ 1, & c_{a_i}^2 \geq 1 \end{cases}, \quad E[W_i]_{M/M/m_i:FCFS} = \frac{(m_i\rho_i)^{m_i} \pi_i(0)}{\mu_i m_i (1-\rho_i)^2 m_i!} \quad \text{and} \quad \pi_i(0) =$$

$$\left\{ \sum_{j=0}^{m_i-1} \frac{(m_i\rho_i)^j}{j!} + \frac{(m_i\rho_i)^{m_i}}{(1-\rho_i)m_i!} \right\}^{-1}. \quad \text{The expected number of jobs at station } i \text{ under the first come first}$$

served (FCFS) service protocol is  $E[N_i]_{GI/G/m_i:FCFS} = \rho_i + \lambda_i E[W_i]_{GI/G/m_i:FCFS}$  and

$E[N_2]_{GI/G/m_i:FCFS} = \lambda_2 E[S_2]$ . For shortest expected processing time protocol (SEPT) time for job

class  $(l,r)$  at station 4,  $E[W_4^{(l,r)}]_{GI/G/1:SEPT} \approx \frac{1-\rho_4}{(1-\rho_4^{(l,r)})(1-\rho_4^{(k,s)})} E[W_4]_{GI/G/1:FCFS}$ , where  $(k,s)$  is the

job class with lower priority next to jobs  $(l,r)$  and  $\rho_4^{(l,r)} = \lambda_4^{(l,r)} E[S_4^{(l,r)}]$ . The expected number of

aggregate jobs at station 4 is  $E[N_4]_{GI/G/1:SEPT} = \rho_4 + \sum_{l,r} \left( \frac{\lambda_4^{(l,r)}(1-\rho_4)}{(1-\rho_4^{(l,r)})(1-\rho_4^{(k,s)})} \right) E[W_4]_{GI/G/1:FCFS}$ .

The formula for the total profit is

$$\begin{aligned} \text{Total Profit} &= \text{Revenue\_New} + \text{Revenue\_Refurbish} + \text{Revenue\_Dismantle} - \text{Transfer Cost} \\ &\quad - \text{Backorder Cost} - \text{Inventory Cost} \\ &= P_{new} \lambda_{new} (1 - p_{cr}^{(1)}) + \sum_{l=2}^{\eta+1} P_{ref} \lambda_4^{(l)} (1 - p_{cr}^{(l)}) + \sum_{l=1}^{\eta} \sum_{r=1}^{\theta} P_{dis}^{(l,r)} \lambda_{30}^{(l,r)} \\ &\quad - \sum_{ij \in A} \sum_{l=1}^{\eta} \sum_{r=1}^{\theta} c_{ij}^{(l,r)} \lambda_{ij}^{(l,r)} - h_1 E[N_1] - \sum_{i=3}^5 h_i E[N_i] \end{aligned}$$

The objective function is nonlinear with multiple local optima with  $P_{ref}$  and  $p_{mr}^{(l,r)}$  as continuous decision variables. To find the optimal solution, we decompose the original problem into several sub problems where it is guaranteed that all local optima will be found. For example, considering the grade-type priority policy where OEMs consider both grades and return histories to decide the proportion of products to be refurbished, the problem is decomposed into  $l \times r$  sub-problems. Sub-problem has two decision variables:  $P_{ref}$  and  $p_{mr}^{(l,r)}$ . Let job classes  $\mathbf{H}$  be the set of job classes with higher priority than  $(l,r)$  and  $\mathbf{L}$  be the set of job classes with lower priority than  $(l,r)$ . Then  $p_{mr}^{(k,s)} = 1, \forall (k,s) \in \mathbf{H}$  and  $p_{mr}^{(m,t)} = 0, \forall (m,t) \in \mathbf{L}$ . For example, let  $\eta = 2, \theta = 3$  and  $\succ$  mean ‘‘has higher priority than’’. Job with different types and grades are sorted according to their time and cost to refurbish as follows:  $(l,r) = (1,1) \succ (2,1) \succ (1,2) \succ (2,2) \succ (1,3) \succ (2,3)$ . We now have six sub-problems. In subproblem (2,2), the decision variable  $0 \leq p_{mr}^{(2,2)} \leq 1$ ,  $\mathbf{H} = \{(1,1), (2,1), (1,2)\}$  and  $\mathbf{L} = \{(1,3), (2,3)\}$ . The (sub) optimal solution from each of these sub-problems will be compared to find the overall optimal solution that maximizes the total profit.

For the grade-type priority problem, let  $f(P_{ref}, p_{mr}^{(l,r)})$  be the total profit function in terms of the decision variables  $P_{ref}$  and  $p_{mr}^{(l,r)}$ ,  $l = 1, \dots, \eta$  and  $r = 1, \dots, \theta$ . The  $(l, r)$  subproblem is

$$\text{Max } f(P_{ref}, p_{mr}^{(l,r)})$$

**Subject to:**

$$P_{ref} \geq P_{new} - (1 - \delta) \quad (1)$$

$$P_{ref} \leq \delta P_{new} \quad (2)$$

$$\rho_i \leq 1 - \varepsilon_i, \text{ for } i = 1, 3, 4, 5 \quad (3) - (6)$$

$$\rho_5 \geq \gamma \quad (7)$$

$$p_{mr}^{(l,r)} \leq 1 \quad (8)$$

$$p_{mr}^{(l,r)} \geq 0 \quad (9)$$

where  $\varepsilon_i$ ,  $i = 1, 3, 4, 5$ , are some small positive constants. Constraints (1) and (2) arise from the demand function to guarantee the nonnegativity of demand for new and refurbished products. Inequalities (3) to (6) are the nonstrict inequality form of steady-state conditions, suitable for optimization software. At station 5 where the demand for refurbished products occurs, constraint (7) assigns a minimum service level,  $\gamma$ , for the refurbished products store. The service level is typically known as one minus the ratio of lost sales to the maximum possible sales. In our model, the maximum possible sales equals  $\lambda_{ref}$  and actual total sales rate of refurbished products is  $\lambda_5$ . Therefore the service level is  $1 - \frac{\lambda_{ref} - \lambda_5}{\lambda_{ref}}$  or essentially the utilization of station 5 which is the ratio of the supply rate to the demand rate of refurbished products,  $\rho_5 = \lambda_5 E[S_5] = \frac{\lambda_5}{\lambda_{ref}}$ . We set  $\gamma > 0$  to assure that when the manufacturer sets the

price of refurbished products to the point that their demand occurs, there are some refurbished products to supply these demands. To guarantee the convexity of the constraint (7),  $\gamma$  is set to a value less than  $\delta$  (see proof in Appendix 1 of [9]). Constraints (8) and (9) are bounds on the probability  $p_{mr}^{(l,r)}$ . Constraints (5) and (6) are immaterial when  $p_{mr}^{(l,r)} = 0$  because there are no arrivals to stations 4 or 5.

In line with [15], we assume that the time to evaluate returned products is short enough that the manufacturer should have ample capacity to handle all returned products; therefore, we choose our parameters in such a way as constraint (4) is automatically satisfied. We also assume that the refurbishing site has enough capacity to process the maximum possible rate of incoming returned products, i.e.,  $\mu_4 > \lim_{p_{mr}^{(l,r)} \rightarrow 1} \lambda_{new} \left( \sum_{k=1}^{\eta} \left( \prod_{l=1}^k p_{cr}^{(l,\bullet)} \sum_{r=1}^{\theta} p_{mr}^{(l,r)} \right) \right)$ , which implies that (5) is less restrictive than (2). It can be shown in turn that (6) renders (2) redundant while (7) is more restrictive than (1). The effect of new product manufacturing capacity has been studied in [8]. In this paper, we further assume that sufficient manufacturing capacity for new products exists to render (3) insignificant. Therefore the feasible region is defined by (6), (7), (8) and (9). The feasible region of sub-problem is convex only when  $\eta \rightarrow \infty$  and no grade or type are considered; otherwise, it is non-convex (see proof in Appendix B of [24]).

The total profit is not pseudoconcave over the entire feasible region. We found that each sub-problem contains at most two local optima. The first local optimum lies at the point where  $p_{mr}^{(l,r)} = 0$ . The other local optimum lies either interior to the feasible region or on the boundary where  $p_{mr}^{(l,r)} = 1$ . When the feasible region is convex, local optima can be found by a hill-climbing procedure from an initial value of  $p_{mr}^{(l,r)}$  between 0 and 1. When the feasible region is not convex, the local optima are verified by comparing them with other points in an  $\varepsilon$ -

neighborhood around the stopping point. Local optima are compared to obtain the globally optimal solution.

### **3. Numerical Results**

We performed numerical experiments to explore the following questions: 1) How are the total profit and optimal solutions affected by variability in the service times at different stations? 2) How sensitive are optimal solutions to the number of times a product may be refurbished versus the return probability? 3) What are the impacts of prioritizing returns for refurbishment based on their return histories and grades?

#### ***3.1 Effect of Service Time Variability***

Vorasayan and Ryan [8] analyzed the optimal price and quantity of refurbished products in several scenarios under the assumptions of exponentially distributed service times (with  $scv$  equal to 1) at each station, no limit to the number of times a product can be refurbished, and no differentiation of returns based on grade. The model reduced to a Jackson queueing network in which each station could be analyzed independently. In reality, previous studies have found that the  $scv$  of the time to assemble a new product is less than 1 while the  $scv$  of remanufacturing processes range from 0.5 to 2 (see [16]). Moreover, because the length of time products are in consumers' possession is limited by the allowed return period, the exponential distribution, with no finite upper limit, is not a good representation of it. In this paper, we relax the assumptions of exponential processing times at stations 1 – 4. However, we retain the assumption of exponential interarrival times of the demands for both new and refurbished products. Note that the latter time intervals are represented by service times at station 5; their memoryless property

enables the simplification that times between demand arrivals are independent of the availability of refurbished products.

To focus solely on the effect of service time variability, we assume job grades and types are aggregated so that the flow rates are exactly the same as in the Jackson network model, the service protocols are FCFS for all stations, and we do not limit the number of times products are refurbished, i.e.,  $\eta \rightarrow \infty$ . When  $C_{s_i}^2 = 1, i = 1, \dots, 5$ , this generalized open queueing network with no priority yields the identical results to the Jackson open queueing network model in [8]. Following are the constant parameter values in this section:  $P_{new} = 0.45, \bar{P}_{dis} = 0.15, p_{cr}^{(l^*)} = 0.25$  for  $l = 0, \dots, \infty, c_{01} = c_{23} = c_{20} = c_{40} = c_{52} = 0, c_{12} = 0.25, c_{30} = 0.02, c_{34} = 0.01, c_{45} = 0.06, h_1 = 0.0001, h_2 = 0, h_3 = 0.00005, h_4 = 0.00005, h_5 = 0.00005, \mu_1 = 0.6, \mu_2 = 0.006, \mu_3 = 0.3, \mu_4 = 0.6$ .

In any queueing model where jobs arrive and are served one at a time, a queue of jobs is induced by variability in the service time. Therefore, the higher variability at stations where the manufacturer pays inventory costs results in less total profit. The scv of service times of a particular station is varied from 0 to 2 while scv of other station are set constant as follows: At the manufacturing station,  $C_{s_1}^2 = 0.25$ , which is in the range of scv of assembly line in [25]. At station 2,  $C_{s_2}^2 = \left( \frac{2E[S_2]^2}{12} \right) / E[S_2]^2 = 0.33$  by assuming that the time a product will be held by the consumer has a uniform distribution on the interval  $[0, 2E[S_2]]$ . The inspection and refurbishment scvs are  $C_{s_3}^2 = 0$  and  $C_{s_4}^2 = 1.7825$  according to the study by [15]. At the store for refurbished products,  $C_{s_5}^2 = 1$  because the service time at station 5 represents the exponentially distributed interarrival time of demands for refurbished products.

Recall that from the previous section, when  $\theta = 1$ ,  $p_{cr}^{(k,\bullet)} = p_{cr}$ ,  $p_{mr}^{(k,r)} = p_{mr} \forall (k, r)$ .

Therefore,  $\lim_{\eta \rightarrow \infty} \lambda_4 = \lim_{\eta \rightarrow \infty} \left\{ \lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k)} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right) \right\} = \left( 1 - \frac{P_{new} - P_{ref}}{1 - \delta} \right) \left( \frac{p_{cr} p_{mr}}{1 - p_{cr} p_{mr}} \right)$ . By

optimizing  $P_{ref}$  and  $p_{mr}$ , we obtain the optimal rate of supply of refurbished items:

$$\lambda_4^* = \left( 1 - \frac{P_{new} - P_{ref}^*}{1 - \delta} \right) \left( \frac{p_{cr} p_{mr}^*}{1 - p_{cr} p_{mr}^*} \right).$$

In Figures 2 and 3, we plot optimal supply curves for refurbished products as  $P_{ref}^*$  against  $\lambda_4^*$  for different levels of service time variability at stations 1 and 4, respectively. They are parameterized by the willingness-to-pay,  $\delta$ . For low values of  $\delta$ ,  $p_{mr}^* = 0$ , i.e., it is optimal not to refurbish any returns. As  $\delta$  increases, the optimal price and supply of refurbished items both increase as long as  $0 < p_{mr}^* < 1$ . For very high  $\delta$ , it is optimal to refurbish all returns, i.e.,  $p_{mr}^* = 1$ . Paradoxically, this reduces the optimal supply of refurbished products. Their price continues to increase with  $\delta$ , but slowly enough that the downward pressure from  $\delta$  on  $\lambda_{new}$  remains stronger than the upward pressure from increasing  $P_{ref}$ . The net effect is to reduce the demand for new products, which in turn diminishes the number of returns available to refurbish.

Higher variability in manufacturing times (at station 1) raises  $\lambda_4^*$  and lowers  $P_{ref}^*$ , i.e., it shifts the product portfolio towards supply of refurbished rather than new products. Shifting demand away from new products is a way to control the lengths of customer queues for them. In contrast, higher variability in refurbishing times lowers  $p_{mr}^*$  and raises  $P_{ref}^*$  to mitigate the higher number of returns waiting to be refurbished otherwise. The variability at station 2 has no effect on the optimal solution because of two reasons: 1) there is no inventory cost at station 2 and 2)

the variability at station 2 with infinite servers does not affect the variability of arrival times to

other stations, i.e., as  $m \rightarrow \infty$ ,  $c_{d_2}^2 = \lim_{\rho_2 \rightarrow 0, m_2 \rightarrow \infty} \left( 1 + (1 - \rho_2^2)(c_{a_2}^2 - 1) + \frac{\rho_2^2(c_{s_2}^2 - 1)}{\sqrt{m_2}} \right) = c_{a_2}^2$ . Similarly,

the effect of variability at station 3 on total profit and  $(P_{ref}^*, p_{mr}^*)$  is very small because the small value of  $\rho_3$  due to selected parameters leads to low inventories and little effect on variability of arrival times to other stations.

As expected, the optimal profit decreases as the processing time variability at either station 1 or station 4 decreases. Therefore, to maximize total profit, OEMs should keep variability at station 1 and 4 as low as possible. The example in section 3.3 illustrates one way to reduce variability at station 4.

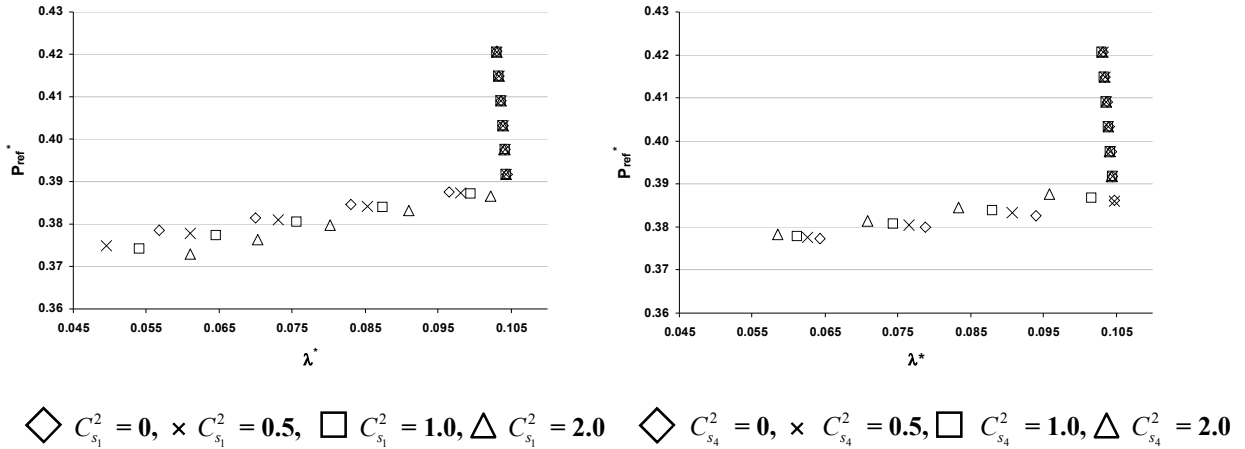


Figure 2. Supply curves based on variability at stations 1 and 4

### 3.2 Repeatability of Refurbishment and Probability of Returns

Next, we study the change in the optimal solution when returned products can be refurbished only a finite number of times. The parameter  $\eta$  is assigned values of 0, 1, 2, 3 and  $\infty$ ,

where  $\eta = 0$  represents a no refurbishment policy. In our example, we assume different grade distributions among new and refurbished products. The returns from brand new products intuitively consist of higher proportion of better grades than those of returned refurbished products. However, we assume that consumers are unable to distinguish how many times a product has been refurbished and, therefore, their willingness-to-pay,  $\delta$ , remains constant. Note that a finite value for  $\eta$  implies that the fraction of returns that has reached its refurbishing limit (type  $\eta + 1$ ) are disposed of automatically. Thus, unlike in [8],  $p_{mr}^* = 1$  here does not mean that all returns are refurbished.

Let product returns have three grades ( $\theta = 3$ ). Based on [2] and [26], we assign  $q^{(1,1)} = 0.4$ ,  $q^{(1,2)} = 0.4$ ,  $q^{(1,3)} = 0.2$  while  $q^{(l,1)} = 0.2$ ,  $q^{(l,2)} = 0.4$ ,  $q^{(l,3)} = 0.4$  for  $l = 2, \dots, \eta + 1$ . We assume that the refurbishing time and cost as well as the price of dismantled products for each grade are uniform over all types, in other words, independent of the number of times the product has been returned previously. Therefore,  $E[S_4^{(l,r)}] = E[S_4^{(\bullet r)}]$ ,  $c_{ij}^{(l,r)} = c_{ij}^{(\bullet r)}$  and  $P_{dis}^{(l,r)} = P_{dis}^{(\bullet r)}$ . Also, the holding and dismantling costs, are independent of both grades and types. These parameter values are assigned as follows  $\bar{P}_{dis}^{(\bullet 1)} = 0.16$ ,  $\bar{P}_{dis}^{(\bullet 2)} = 0.15$ ,  $\bar{P}_{dis}^{(\bullet 3)} = 0.13$ ,  $c_{34}^{(\bullet 1)} = 0.04$ ,  $c_{34}^{(\bullet 2)} = 0.06$ ,  $c_{34}^{(\bullet 3)} = 0.10$ ,  $E[S_4^{(\bullet 1)}] = 2.5$ ,  $E[S_4^{(\bullet 2)}] = 3.5$ ,  $E[S_4^{(\bullet 3)}] = 4.67$ .

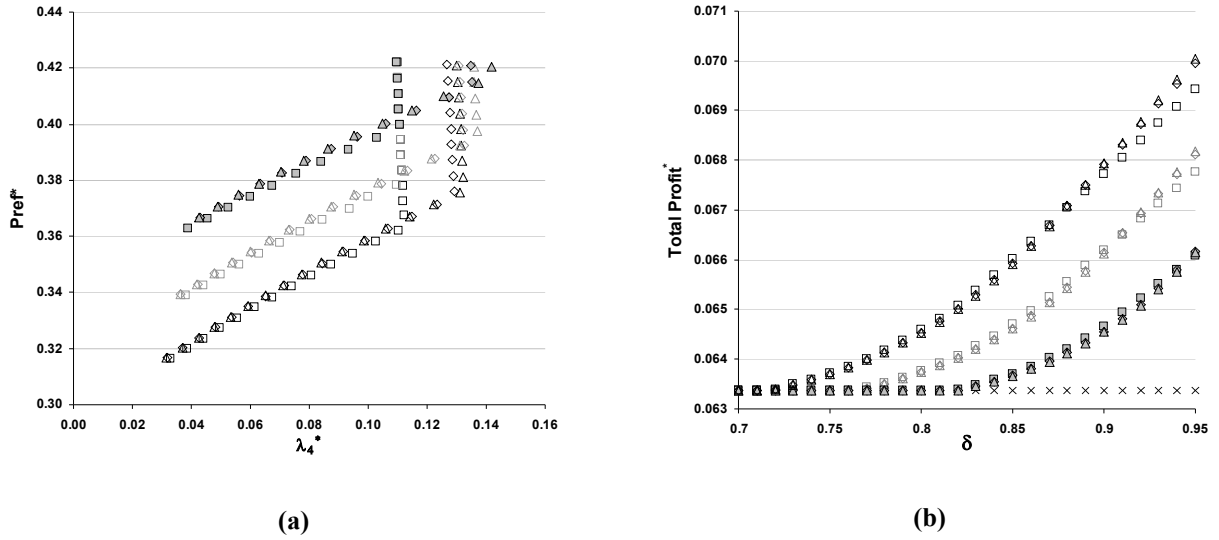
Moreover, although warranty and support policies of new products in the same category are similar, electronic manufacturers<sup>1</sup> provide different return policies for their refurbished products. These policies tend to affect the probability of returns. We assume the same probability of returns for new products  $p_{cr}^{(1\bullet)} = 0.25$  and three different probabilities of returns for previously

---

<sup>1</sup> Apple provides a one year limited warranty while HP allows only three months for their refurbished notebooks (see <http://www.apple.com> and <http://www.hp.com>).

refurbished products  $p_{cr}^{(l^*)} = 0.2, 0.25, 0.3$  for  $l = 2, \dots, \eta + 1$ . The other parameters are the same as in the previous section.

Recall that  $\lambda_4 = \lambda_{new} \left( \sum_{l=1}^{\eta} \prod_{k=1}^l p_{cr}^{(k^*)} \sum_{r=1}^{\theta} (p_{mr}^{(k,r)} q^{(k,r)}) \right)$ . Therefore, in this example  $\lambda_4^* = \left( 1 - \frac{P_{new} - P_{ref}^*}{1 - \delta} \right) \left( \sum_{l=1}^3 \prod_{k=1}^l p_{cr}^{(k^*)} \sum_{r=1}^3 (p_{mr}^{(k,r)^*} q^{(k,r)}) \right)$ .



- $\square$   $p_{cr}^{(l^*)} = 0.20, \eta = 1$ ,  $\diamond$   $p_{cr}^{(l^*)} = 0.20, \eta = 2$ ,  $\triangle$   $p_{cr}^{(l^*)} = 0.20, \eta = 3$ ,
- $\square$   $p_{cr}^{(l^*)} = 0.25, \eta = 1$ ,  $\diamond$   $p_{cr}^{(l^*)} = 0.25, \eta = 2$ ,  $\triangle$   $p_{cr}^{(l^*)} = 0.25, \eta = 3$ ,
- $\square$   $p_{cr}^{(l^*)} = 0.30, \eta = 1$ ,  $\diamond$   $p_{cr}^{(l^*)} = 0.30, \eta = 2$ ,  $\triangle$   $p_{cr}^{(l^*)} = 0.30, \eta = 3, l = 1, 2, 3$ ,  $\times$  No refurbishing

**Figure 4. (a)  $P_{ref}^*$  vs  $\lambda_4^*$ , (b) Total profit for the different refurbishabilities and probabilities of refurbished product returns.**

We optimized the objective function at fixed willingness-to-pay levels,  $\delta$ , from 0.7 to 0.95. This parameter is one of the key factors that affect profitability and the decision of the proportion of returned products to be refurbished [8]. Figure 4 shows the supply curves for different return probabilities and refurbishing limits parameterized by  $\delta$  along with the optimal total profit as a function of  $\delta$ . The optimal proportion of returned products to be refurbished is

affected largely by the return probability,  $p_{cr}^{(l^*)}$ . A lower return probability of previously refurbished products results in higher proportion of them being refurbished. Overall, increasing the return probability of previously refurbished products shifts the supply curve up- and leftward, i.e. toward higher price and lower supply of refurbished products. This effect can be explained by reference to the total profit function. Because the sale price is refunded when the product is returned, larger values of  $p_{cr}^{(l^*)}$  for  $l \geq 2$  directly reduce the net revenue from selling refurbished products. This effect can be mitigated partially by increasing  $P_{ref}$ , which also shifts demand toward new products. The supply of refurbished products is lowered to match the lower demand. Interestingly, at some combinations of probability of return and willingness-to-pay for refurbished products, the maximum total profit is achieved when finite limits are imposed on the number of times a product may be refurbished. The results in Table 2 show the cases where allowing refurbishment only once or twice yields higher profit than refurbishing without limit or not at all. When the willingness-to-pay for refurbished products is low and the return probability is high, it is optimal to not refurbish or allow refurbishing only once or twice.

**Table 2. Ranking of  $\eta$  in order to their optimal total profit for different  $p_{cr}^{(l^*)}$  and  $\delta$**

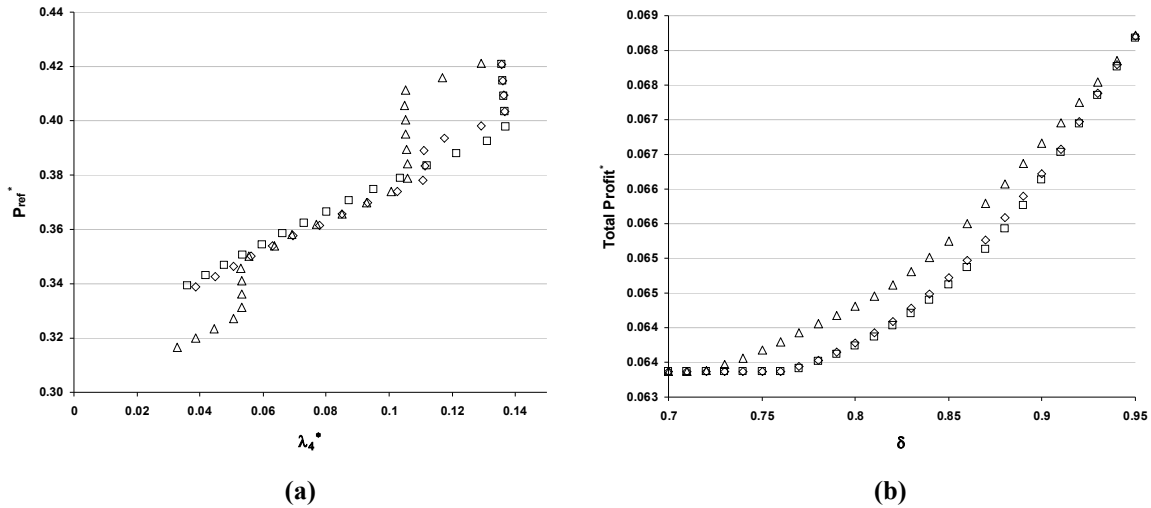
$p_{cr}^{(l^*)}$	$\delta$					
	0.70	0.75	0.80	0.85	0.90	0.95
<b>0.20</b>	optimal not to refurbish	$1 > 2 > 3 > \infty > 0$	$1 > 2 > 3 > \infty > 0$	$1 > 2 > 3 > \infty > 0$	$\infty > 3 > 2 > 1 > 0$	$\infty > 3 > 2 > 1 > 0$
<b>0.25</b>	optimal not to refurbish	optimal not to refurbish	$1 > 2 > 3 > \infty > 0$	$1 > 2 > 3 > \infty > 0$	$1 > 2 > 3 > \infty > 0$	$\infty > 3 > 2 > 1 > 0$
<b>0.30</b>	optimal not to refurbish	optimal not to refurbish	optimal not to refurbish	$1 > 2 > 3 > \infty > 0$	$1 > 2 > 3 > \infty > 0$	$2 > 3 > \infty > 1 > 0$

### 3.3 Priority for Sorted Returned Products

The refurbishing priority can be based on type (i.e., return history) or grade or both. For our parameter settings where costs and price are independent of the number of times a product has been returned, the optimal solution found with grade priority is the same as with priority on both grade and type. Therefore, in Figure 5 we show the optimal supply curves of refurbished products and total profits based on no priority, type-only priority and grade-only priority for  $\eta = 3$ . Priority is given to jobs that require less cost and time to be refurbished. Therefore, the type priority order is type 1  $\succ$  type 2  $\succ$  type 3 and the grade priority order is grade 1  $\succ$  grade 2  $\succ$  grade 3. These priority orders are equivalent to Shortest Expected Processing Time (SEPT), which yields higher total profit than a FCFS policy due to fewer jobs in the queue at station 4.

The results illustrate how sorting and prioritizing returned products can improve total profit. The differences of improvement are significant when we refurbish only some portion of returned products and decrease as the proportion of returns that are refurbished increases. When refurbishing all of the returns, there is no difference between no priority and any priority policy. Priority on grade gives the highest total profit in this example because of a significant cost and inventory saving. Priority on type alone also shows a modest improvement in total profit over no priority because returned products from first time usage have better grade distributions than those that have been refurbished previously. The price is adjusted according to the quantity. At low values of  $\delta$ , it is more profitable to refurbish than to sell new products only when the best grade of returned products are refurbished. As the willingness-to-pay increases, refurbishing is a more profitable option but then lower-grade returns products still make less profit. The vertical jumps in  $P_{ref}^*$  occur when there are only slight changes in  $\lambda_4^*$  compared to  $P_{ref}^*$  as a result of a constant proportion of products to be refurbished. For example, in the grade-priority case, when

$P_{ref}^*$  increases from 0.33 to 0.345,  $\lambda_4^*$  decreases only from 0.0532 to 0.0530. At this range, the proportion of refurbishing grade one is 100% but 0% for grade 2 because profit margin in grade two makes refurbishing not worthwhile. But increasing  $P_{ref}^*$  reduces returns which constitute the source of supply. This is why  $\lambda_4^*$  decreases slightly as  $P_{ref}^*$  goes up. This effect is more potent in the grade-priority case than the type-priority case.



□ No priority, ◇ Priority on Type, △ Priority on Grade

Figure 5. (a)  $P_{ref}^*$  vs  $\lambda_4^*$ , (b) Total profit for no priority, priority on type and priority on grade.

#### 4. Conclusion and Future Research

To find the best policy to manage returned products, there are several factors to consider: the consumer willingness-to-pay for refurbished products, variability in the manufacturing and refurbishment times, probabilities of products being returned and the number of times products can be repeatedly refurbished. Changes in parameters would change the optimal profit as well as the optimal supply curve for offering refurbished products. The general queueing model

efficiently captures many aspects of the closed-loop supply chain and allows us to see the benefit of sorting and prioritizing returned products and the effects of these efforts on the total profit and optimal decisions. It also allows accurate representations of the variability of the processing time and tracking of return histories to analyze their effects.

From the numerical results with realistic values for the exogenous parameters, the variability in manufacturing or refurbishing times contributes to backorder and inventory costs that negatively affect the profitability. Variability in manufacturing time encourages the offering of more refurbished products at a lower price, while variability in the time to refurbish returns has the opposite effect. The number of times products should be repeatedly refurbished is not explicitly a decision variable in this model, but is another important attribute that affects product design. When the price consumers are willing to pay for refurbished products is high enough that refurbishing is profitable but still significantly less than the new product price, maximum profit is achieved if products can be refurbished at most once. The proportion of previously refurbished products that are returned (i.e., accepted) again has a pronounced negative impact on profit and causes an optimally smaller number of refurbished products to be offered at a higher price.

When quality grades of returned products are distinguishable, sorting and prioritizing returns reduces the cost and time for refurbishing. These savings are significant when the manufacturer refurbishes only some of returns. However, identifying grades of returned products might require more time and cost than simply tracking return history. In our numerical example, the grade distribution was invariant over the return history, but the proposed model is capable of prioritizing based on both grade and return history. In more complex scenarios when costs and

processing times of refurbishing products varies by both attributes, prioritizing on both will yield results superior to prioritizing solely on either one.

We believe the application of our model is not limited to only electronics manufacturing, but can be applied in a variety of industries where products are manufactured to order and refurbished to stock. As observed recently many companies such as General Electric, American Standard, BMW, and Toyota are moving toward the make-to-order approach [27]. Moreover, automotive industries such as Honda, BMW and Toyota are already selling their certified pre-owned vehicles online.

Dell now reveals more information about their previously owned products before reselling them to consumer. Products are grouped into three categories: 1) previously ordered new 2) certified refurbished and 3) scratch and dent. This practice allows the consumer more freedom to choose according to their personal preferences. The amount of information revealed and number of categories of pre-owned products may be interesting topics for future study.

In our queueing system, facilities for producing new and refurbished products operate independently. Realistically, OEMs might use the same facility and manpower to process both products. The steady state assumption in this paper is applicable only during the interval of time when market size is constant. The model might not be suitable for the beginning or the end of the product life cycle where demand and market size changes rapidly. It can be improved by assigning parameters such as market size, price and cost as functions of time.

## **Appendix A**

The notations are summarized as follows:



$c_{ij}$	Cost of transferring jobs from station $i$ to $j$
$c_{a_i}^2$	Squared coefficient of variation (scv) of the external interarrival time at station $i$
$c_{s_i}^2$	Scv of the service time of aggregate jobs at station $i$
$h_1$	Backorder cost of new product
$h_i$	Holding cost of finished products at station $i, i = 3, 4, 5$
$p_{cr}^{(l^*)}$	The probability of products type $l$ (independent of their grade) being returned by consumer
$p_{ij}^{(k)(l)}$	The probability that a job leaving station $i$ as a type $k$ job will join station $j$ as a type $l$ job, $ij \in A$
$p_{ij}$	The transfer probability of aggregate jobs from station $i$ to station $j, ij \in A$
$p_{mr}^{(l,r)}$	The probability that the OEM will send products class $(l,r)$ to be refurbished
$p_{mr}^{(l^*)}$	The probability that the OEM will send an $l^h$ -return of product (any grade) to be refurbished
$p_{mr}^{(r^*)}$	The probability that the OEM will send a product of grade $r$ (any type) to be refurbished
$P_{new}$	Price of selling new products
$P_{ref}$	Price of selling refurbished products
$P_{dis}^{(l,r)}$	Price of selling dismantling product class $(l,r)$
$\bar{P}_{dis}^{(l,r)}$	Starting price of selling dismantling product class $(l,r)$
$q^{(l,r)}$	The proportion of jobs class $(l, r)$
$E[S_i^{(l,r)}]$	Expected service time of jobs class $(l, r)$ at station $i$
$E[S_i]$	Expected service time of aggregate jobs at station $i$
$E[W_i]$	The expected waiting time of aggregate jobs at station $i$
$E[N_i]$	The expected number of jobs at station $i$
$\lambda_{new}$	Demand of new products
$\lambda_{ref}$	Demand of refurbished products
$\lambda_{ij}$	Flow rate of aggregated jobs from station $i$ to station $j$
$\lambda_{ij}^{(l,r)}$	Flow rate of jobs type $l$ grade $r$ or class $(l, r)$ from station $i$ to station $j$
$\delta$	Perceived willingness-to-pay of refurbished products
$\theta$	Number of grades of returned products
$\mu_i$	Processing time at station $i$

## Appendix B

For probabilistic routing [28], the scv of departures or arrivals of aggregate jobs that move from station  $i$  to station  $j$  with probability  $\hat{p}_{ij}$  is  $c_{d_{ij}}^2 = c_{a_{ij}}^2 = \hat{p}_{ij}c_{d_i}^2 + 1 - \hat{p}_{ij}$ . On the other hand, for deterministic routing, Whitt [21] proposes the scv of departures of type  $l$  at station  $i$  as  $c_{d_i}^{2(l)} = d_i^{(l)}c_{d_i}^2 + (1 - d_i^{(l)})d_i^{(l)}\bar{c}_{a_i}^{2(l)} + (1 - d_i^{(l)})^2c_{a_i}^{2(l)}$  where  $d_i^{(l)}$  is the proportion of all departures at station  $i$  that are type  $l$ ,  $c_{a_i}^{2(l)}$  is the scv of the arrival of type  $l$  and  $\bar{c}_{a_i}^{2(l)}$  is the scv of the arrival of all types except type  $l$ , given by  $\bar{c}_{a_i}^{2(l)} = \frac{c_{a_i}^2 - \frac{\lambda_i^{(l)}}{\lambda_i}c_{a_i}^{2(l)}}{\left(1 - \frac{\lambda_i^{(l)}}{\lambda_i}\right)}$ . As discussed above, all jobs types that flow from station  $i$  to station  $j$  are treated similarly, therefore, we simplify the process by assuming  $c_{d_i}^2 \approx c_{a_i}^{2(l)}$ . Then the above equation can be rewritten in terms of  $c_{d_{ij}}^2$  of aggregate jobs such that  $c_{d_{ij}}^2 = \hat{d}_{ij}c_{d_i}^2 + (1 - \hat{d}_{ij})c_{a_i}^2$  where  $\hat{d}_{ij}$  is transfer probability of aggregate job from  $i$  to  $j$  from deterministic routing. In our case,  $\hat{d}_{ij} = 1$  for  $ij = \{01, 12, 45, 52\}$ .

The hybrid approximation is the convex combination of both  $c_{d_{ij}}^2$  from probabilistic and deterministic routings, found by  $c_{d_{ij}}^2 = (1 - \beta_{ij})(\hat{p}_{ij}c_{d_i}^2 + 1 - \hat{p}_{ij}) + \beta_{ij}(\hat{d}_{ij}c_{d_i}^2 + (1 - \hat{d}_{ij})c_{a_i}^2)$  where  $\hat{d}_{ij} \in \{p_{01}, p_{12}, p_{45}, p_{52}\}$ ,  $\hat{p}_{ij} \in \{p_{20}, p_{23}, p_{30}, p_{34}\}$  and  $\beta_{ij}$  is the proportion of all flow from  $i$  to  $j$  that is due to deterministic routing as opposed to probabilistic routing [20].

## References

- [1] Toktay, L. B., van der laan E., Brito M. P. Managing Product Returns: The Role of Forecasting, Econometric Institute Report 2003; EI 2003-11.
- [2] Guide V. D. Jr., Souza G. C., Van Wassenhove L. N., Blackburn J. D. Time Value of Commercial Product Returns, *Management Science* 2006; 52(8): 1200-1214.
- [3] Guide, V. D. Jr. Production Planning and Control for Remanufacturing: Industry Practice and Research Needs. *Journal of Operations Management* 2000; 18: 467-483.
- [4] Guide V. D. Jr., Teunter R. H., Van Wassenhove L. N. Matching Demand and Supply from Remanufacturing. *Manufacturing and Service Operations Management* 2003; 5(4): 303-316.
- [5] Thierry, M.C., Salomon M., Van Nunen J., Van Wassenhove L. N. Strategic production and operations management issues in product recovery management, *California Management Review* 1995; 37(2): 114-135.
- [6] Debo L.G., Toktay L.B., Van Wassenhove L. N. Market Segmentation and Production Technology Selection for Remanufacturable Products, *Management Science* 2005; 51(8): 1193-1205.
- [7] Ferguson M., Toktay L. B. The Effect of Competition on Recovery Strategies. *Production and Operations Management* 2006; 15(3): 351-368.
- [8] Vorasayan J., Ryan S. M. Optimal Price and Quantity of Refurbished Products, *Production and Operations Management* 2006; 15(3): 369-383.
- [9] Ferrer G., Swaminathan J. M. Managing New and Remanufactured Products. *Management Science* 2006; 52: 15-26.
- [10] Bayindir Z. P., Erkip N., Güllü R. Assessing the benefits of remanufacturing option under one-way substitution and capacity constraint. *Computers and Operations Research*. 2007; 34: 487-514.

- [11] Guide V. D. Jr., Jayaraman V., Srivastava R., Benton W. C. Supply-Chain Management for Recoverable Manufacturing Systems. *Interfaces* 2000; 30: 125-142.
- [12] Guide V. D. Jr., Gupta S. M. A Queueing Network Model for a Remanufacturing Production System. *Proceedings of the Second International Seminar on Reuse* 1999: 115-128.
- [13] Toktay L. B., Wein L. M., Zenios S. A. Inventory Management of Remanufacturable Products. *Management Science* 2000; 46(11): 1412-1426.
- [14] Bayindir Z. P., Erkip N., Güllü R. A Model to Evaluate Inventory Costs in a Remanufacturing Environment. *International Journal of Production Economics* 2003; 81-82: 597-607.
- [15] Souza G. C., Ketzenberg M. E. Guide V. D. Jr. Capacitated Remanufacturing With Service Level Constraints. *Production and Operations Management* 2002; 11: 232-248.
- [16] Ketzenberg M. E., Souza G. C. Mixed Assembly and Disassembly Operations for Remanufacturing. *Production and Operations Management* (2003); 12(3): 321-335.
- [17] Lieckens K, Vandaele N., Reverse Logistics Network Design: the Extension Towards Uncertainty. *Computers and Operations Research* 2007; 34: 395 - 416.
- [18] Majumder P., Groenevelt H. Competition in Remanufacturing. *Production and Operations Management* 2001; 10(2): 125-141.
- [19] Buzacott J. A., Shanthikumar J.G. *Stochastic Models of Manufacturing Systems*, Prentice Hall, Englewood Cliffs, NJ., 1993
- [20] Segal M., Whitt W. A Queueing Network Analyzer for Manufacturing. *Annals of Operations Research* 1989; 48: 221-248.

- [21] Whitt, W. Towards Better Multi-class Parametric-Decomposition Approximations for Open Queueing Networks. *Annals of Operations Research* 1994; 48: 221-248.
- [22] Bitran R. G., Morabito R. Open Queueing Networks: Optimization and Performance Evaluation Models for Discrete Manufacturing Systems. *Production and Operations Management* 1996; 5(2): 163-193.
- [23] Bitran R. G., Tirupati D. Multiproduct Queueing Networks with Deterministic Routing: Decomposition Approach and the Notation of Interference, *Management Science* 1988; 34(1): 75-100.
- [24] Vorasayan J. A General Queueing Network Model to Optimize Refurbishing Policies for Returned Products, PhD. Dissertation 2006. Iowa State University.
- [25] Knott, K. A Study of Work-Time Distributions on Unpaced Tasks. *IIE Transactions* 1987; 3: 50-55.
- [26] Ferguson M., Guide V. D. Jr., Souza G. C. Supply Chain Coordination for False Failure Returns, *Manufacturing & Service Operations Management* 2006; 8(4): 376-393.
- [27] Plambeck E. L., A. R. Ward (2005), Optimal Control of High-Volume Assemble-to-Order Systems. Working paper, Stanford University.
- [28] Whitt, W. The Queueing Network Analyzer. *The Bell System Technical Journal*, 1983; 62(9): 2779-2815.