1. Determine whether the following functions are analytic. Discuss whether they have any singular points or if they are entire:
   
   (a) \( \tan(z) \)
   (b) \( e^{\bar{z}} \)
   (c) \( \frac{z}{z^4+1} \)

2. Consider the following complex potential:
   
   \[ \Omega(z) = -\frac{1}{2\pi z}, \]

   referred to as a “doublet”. Calculate the corresponding velocity potential, stream-function, and velocity field. Sketch the streamfunction.

3. Derive the Cauchy-Riemann conditions in polar coordinates:
   
   \[ u_r = \frac{1}{r} v_\theta \quad \text{and} \quad v_r = -\frac{1}{r} u_\theta. \]

4. Suppose that \( f(z) = u(r, \theta) + iv(r, \theta) \) is analytic in a domain \( D \) not containing the origin. Using the Cauchy-Riemann conditions in polar coordinates show that \( u(r, \theta) \) satisfies Laplace’s equation in polar coordinates:
   
   \[ r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0. \]

5. Suppose that \( f(z) = u(r, \theta) + iv(r, \theta) \) is analytic in a domain \( D \) not containing the origin. Given \( u(r, \theta) \), find \( v(r, \theta) \):
   
   (a) \[ u(r, \theta) = r^3 \cos(3\theta). \]
   (b) \[ u(r, \theta) = \frac{1}{r^2} \left( 10r^2 - \sin(2\theta) \right). \]

6. Consider the function
   
   \[ f(z) = z + \frac{1}{z}. \]

   Plot the level curves \( u(x, y) = 0 \) and \( v(x, y) = 0 \). Verify that where these contours intersect they are orthogonal.