Math 267 - Practice exam 2

Problem 1 A solution of 10% perchlorate in water flows at a rate of 8 L/min into a tank holding 200L pure water. The solution is kept well stirred and flows out of the tank at a rate of 6 L/min.

a) Determine the volume of perchlorate in the tank after \( t \) minutes.
b) When will the percentage of perchlorate in the tank reach 6%?

Problem 2 A garage with no heating or cooling has a time constant of 2 hours. The outside temperature varies as a sine wave,

\[ M(t) = 65 + 15 \sin(\pi(t - 8)/12). \]

Determine the time when the building reaches its lowest and its highest temperature, assuming that the exponential term has died off.

Problem 3 A parachutist whose mass is 75 kg drops from a helicopter hovering 2000 meters above the ground. She falls to the ground under the force of gravity and air resistance, which is proportional to her velocity, with proportionality constant \( b_1 = 30 N/sec/m \) when the chute is closed, and \( b_2 = 90 N/sec/m \) when the chute is open. The chute does not open until the velocity reaches 20 m/sec. At what time will she reach the ground?

Problem 4 Find the general solution of

\[ y'' - 2y' + 5y = 0. \]

Problem 5 Consider the differential equation

\[ x''(t) - 2x'(t) + x(t) = \cos t. \] (1)

a) Find a particular solution of equation (1).
b) Find the complimentary function (general solution of the associated homogeneous equation).
c) Find the unique solution satisfying both (1) and the initial conditions $x(0) = 0$, $x'(0) = 1$.

**Problem 6** A mass of 2 kg is traveling horizontally on wheels without friction. It is hooked up to a spring with spring constant $k = 162$ N/m. Let $x(t)$ be the position of the mass with $x = 0$ being the equilibrium position.

a) Find the second-order linear differential equation governing $x$.

b) What is the natural frequency $\omega$ of this system?

c) Find the position function $x(t)$ if $x(0) = 12$ and $x'(0) = -45$.

d) Suppose an external force $F = 2 \cos(10t)$ starts acting on the mass, and $x(0) = x'(0) = 0$. Find the resulting position function $x(t)$.

**Problem 7** Consider the Euler-Cauchy equation

$$t^2 y'' + 3ty' + y = \frac{1}{t}.$$ 

a) Find the general solution.

b) Then find the solution satisfying $y(1) = 4$, $y'(1) = 3$. 

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