Center of Mass – Gall, Graham, Wilt

Situation
Many double integrals are hard or even impossible to evaluate. Just as for simple (one-dimensional) integrals, Riemann sums are used to define them, and can be used to approximate them as well.

From the summary
Our project was to calculate the center mass of the solid given by
\[ 0 \leq y \leq 4 - x^2 \]
and \[ 0 \leq z \leq e^{(-xy/2)} \]
We calculated the center of mass using MatLab since it is an unsolvable integral by hand.

Thirsty Hiker (Steepest Descent) – Useche Reyes, Mickelson, Zhang

Situation
A thirsty hiker is lost on a mountain slope in dense fog. Unable to see more than a few feet, she/he will instinctively follow the direction of steepest descent to find water.

From the summary
We investigated the function \( f(x, y) = 10x^2 + x + 2xy + y^2 \) to determine the pathway of steepest decent to get to the minimum point of the surface function. We started at three points and used MATLAB to take steps in the direction of the most negative gradient, which is the direction of the steepest decent.

Original plot from the project, but rotated and colored differently.
Situation
A basketball is shot from somewhere on the field with a given direction and speed. Neglecting air resistance, the ball will follow a curve with a quadratic equation. And the question is of course, will it go in or not?

From the summary
The table below is an assortment of scoring shots found by trying random shots and then using the plot to make adjustments.

<table>
<thead>
<tr>
<th>Initial X</th>
<th>Initial Y</th>
<th>Initial Speed</th>
<th>Initial Angle</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0</td>
<td>25.1</td>
<td>45</td>
<td>Free Throw</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>30</td>
<td>73</td>
<td>Free Throw</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>30</td>
<td>40</td>
<td>Three point shot straight out from the basket</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>35</td>
<td>71</td>
<td>Three point shot straight out from the basket</td>
</tr>
<tr>
<td>22</td>
<td>6</td>
<td>31</td>
<td>60</td>
<td>Three point shot from an angle(rebounded)</td>
</tr>
<tr>
<td>47</td>
<td>5</td>
<td>41</td>
<td>45</td>
<td>Half court shot from an angle(rebounded)</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>25</td>
<td>60</td>
<td>Rebounded</td>
</tr>
</tbody>
</table>
Oozing Lava

**Situation**

Lava is oozing down a mountain with height given by \( z = M(x,y) \).

- Project A uses \( M(x,y) = x^2 - 4xy + 2y + 2y^3 \).
- Project B uses \( M(x,y) = x^3 + 2xy + y^2 \).

We assume the lava always moves in the direction where \( M \) decreases most rapidly.

You are a mathematician helping a surveyor and your task is to represent the entire flow of the lava in a given region.

**Lava A – Brown, Mandernach, Mueller, Wiedemeier**
Part a) In every step, you move at a constant distance in the X, Y Plane.

Part c) The 3D - distance between steps is kept approximately constant. As you can see the flatter the slope, the farther in the xy plane the point must move to keep the same 3D distance. The point with 0 gradient is obvious, and lowering the step size gets rid of the zigzags, but clutters up the rest of the plot.
Penalty Method for Constrained Optimization

**Situation**
The goal is to minimize $f(x,y)$ under some constraint $g(x,y)=k$. There is the method of Lagrange Multipliers, or you may be able to parametrize the curve $g(x,y)=k$. An alternative is the penalty method, which minimizes $f$ under a penalty for violating the condition $g=k$, i.e., it minimizes

$$H(x,y) = f(x,y) + \lambda (g(x,y)-k)^2.$$ 

If the minimum for $H$ still violates the constraint condition, $\lambda$ is increased (a stiffer penalty is imposed) and the method repeated.

**OPTCONST B – Cao, Dejmal, Pahl**
The goal is to minimize $f(x,y) = \exp(x^2+3xy-y^2)$ on the ellipse $x^2+4y^2=1$. A quiverplot for $f$ is combined with a contour plot for $g = x^2+4y^2$ and the steps taken by the penalty method.