Math 181 - Quiz 2B, quadratic models - solutions

Problem 1  An oil spill grows over time. The area at time of the $t$-th measurement is $A_t$ in square miles. Suppose $A_0 = 4$, $A_1 = 7$, $A_2 = 19$ and $A_t = at^2 + bt + c$, with constants $a, b, c$. Find the constants $a, b, c$.

First, look at the differences from one measurement to the next:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$A_{t+1} - A_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 - 4</td>
</tr>
<tr>
<td>1</td>
<td>19 - 7</td>
</tr>
</tbody>
</table>

We fit a straight line $y = mx + y_0$ to the two points $(0, 3)$ and $(1, 12)$. The slope of this line has to be

$$m = \frac{12 - 3}{1 - 0} = 9.$$ 

The constant term (some people call it intercept) $y_0$ can be computed from

$$y = 9x + y_0$$

and substituting $x = 0, y = 3$, so $y_0 = 3$. You could also substitute $x = 1$ and $y = 12$, getting the same answer. Now we know that

$$A_{t+1} - A_t = 2at + (a + b)$$

so we equate $9 = 2a$ and $3 = a + b$, so $a = 4.5$ and $b = -1.5$. Finally, from

$$A_0 = c$$

we get $c = 4$, so we have found all our constants. We could write

$$A_t = 4.5t^2 - 1.5t + 4.$$