Math 181 - Practice exam 1

The real exam will be shorter, but there will be plenty of space on the exam paper.

**Problem 1** A population $P_t$ grows exponentially with an initial population $P_0 = 50$ and a doubling time of 10 years ($t$ is the time in years).

a) Find a formula for $P_t$.
b) Find the time when the population has reached 500 units (round the answer to one digit after the decimal point).

**Problem 2** The concentration $A(t)$ of radioactive Carbon 14 in a piece of burnt wood decays exponentially according to

$$A(t+1) - A(t) = -0.000121A(t).$$

a) Find the half-life of Carbon 14.
b) The piece of wood contains now a concentration $A(t) = 0.2A(0)$, where $A(0)$ is the natural concentration of Carbon 14 in living matter. Find the time $t$ that has elapsed since time $t = 0$, when the wood was burned (assume it was the same year that the tree was felled).

**Problem 3** Suppose a deer population $P_t$ increases by 15 percent each spring. Each fall, 750 deer are harvested. At time $t = 0$, there are 3000 deer (time is measured in years).

a) Give a difference equation for the population change.
b) Find the equilibrium for this situation.
c) Find a formula for $P_t$ as a function of time $t$.
d) Estimate the population after 4 years.
e) Will this population eventually go extinct?

**Problem 4** Write $S$ for the stride length of a person. $S$ is roughly a linear function of height $H$. Given the following data points, find the equation for $S$ in terms of $H$ which gives a line passing through both points.

<table>
<thead>
<tr>
<th>$H$</th>
<th>65</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>38</td>
<td>45</td>
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</tbody>
</table>
**Problem 5** Some large birds can only take flight by jumping off a cliff. Suppose such a bird jumps at time $t = 0$, and as long as it has not yet started to flap its wings, its height above water at time $t$ is a quadratic function of the form

$$h(t) = at^2 + bt + c.$$ 

Suppose we have measurements

<table>
<thead>
<tr>
<th>$t$ (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (feet)</td>
<td>200</td>
<td>182</td>
<td>132</td>
</tr>
</tbody>
</table>

a) Find a linear model of time $t$ for the difference $h(t + 1) - h(t)$.
b) Find the equation of $h(t)$.
c) At what time will the bird hit the water if it never starts flapping its wings?

**Problem 6** Find the natural domain of the following functions (describe in terms of equations, inequalities, and/or words).

a) $f(x) = \sqrt{x^2 - 4}$
b) $g(x) = x/(x^2 - x)$.
c) $h(t) = 2t + \log(t + 1)$.

**Problem 7** Sketch a) one graph which is simple, and b) another which is not simple. Make it obvious, please!

**Problem 8** Which of the following are polynomials?

a) $x^3 + \pi x - 0.5$,
b) $x^{3.5} - 1$,
c) $\sqrt{x^2 - 1} + x$,
d) $(\log(x) + 5)^4$. 