Problem 1

Indicate the following from looking at the graph of \( f(x) \) above. All answers are small integers, \( \pm \infty \), or DNE for 'does not exist'.

a) \( \lim_{x \to -1^-} f(x) = \)
b) \( \lim_{x \to -1^+} f(x) = \)
c) \( \lim_{x \to 1^-} f(x) = \)
d) \( \lim_{x \to 1^+} f(x) = \)
e) \( \lim_{x \to 2} f(x) = \)
g) \( \lim_{x \to 3^-} f(x) = \)
h) \( \lim_{x \to 3^+} f(x) = \)
i) All points where \( f(x) \) is not continuous are \( x = \).

Problem 2

Find the equation of the tangent line to the graph of \( f \) at the point \( (1, 0) \), where \( f(x) = x \ln x \).

Problem 3

Find \( f'(x) \) where \( f(x) = \frac{e^x}{x+1} \).

Problem 4

Find \( \frac{dy}{dx} \) where \( y = 2x^3(4x - 3)^5 \).

Problem 5

In a certain production facility, the cost function is

\[ C(x) = 2x + 5 \]
and the revenue function is

\[ R(x) = 8x - x^2 \]

where \( x \) is the number of units produced and sold, and \( R \) and \( C \) are measured in millions of dollars. Find the following.

a) the marginal revenue
b) the marginal cost
c) the break-even point(s) [the number(s) \( x \) for which \( R(x) = C(x) \)]
d) the number \( x \) for which the marginal revenue equals the marginal cost.
e) For which value of \( x \) does the facility make maximal profit?

**Problem 6** The cost of making \( x \) units of a product is \( C(x) = x \ln(x + 5) \).
a) Find the marginal cost of making 100 units.
b) How much will the cost increase approximately, compared to part a), if 4 extra units are made? Use approximation and the marginal cost – the exact value will not get any credit.

**Problem 7** If a rock falls from a height of 40 meters on the planet Jupiter, then its height \( H \) after \( t \) seconds is approximately

\[ H(t) = 40 - 10t^2 \]

a) What is the average velocity of the rock from \( t = 0 \) to \( t = 1 \)?
b) What is the instantaneous velocity at time \( t = 1 \)?
c) What is the acceleration of the rock?
d) When does the rock hit the ground?

**Problem 8** The marginal profit from selling \( x \) units of Hokum is given by two formulas: for \( 0 \leq x \leq 5 \), it is

\[ P'(x) = 12 \]
and for $5 < x \leq 7$, it goes down to

$$P'(x) = 20 - 0.4x^2$$

Given that $P(0) = 0$, find the profit from selling 7 units of Hokum.

**Problem 9** Consider the graph of the function $f(x) = x^3 - 6x^2 - 36x + 1$.

a) Find the intervals on which $f$ is increasing.

b) Find the intervals on which $f$ is decreasing.

c) Find all local maxima of $f$.

d) Find all local minima of $f$.

**Problem 10** What is the slope of the tangent line to the curve given by $xy^2 - x^2y = 2$ at the point $(1, 2)$?

**Problem 11** Find the absolute maximum and absolute minimum values of the function $f(x) = x\sqrt{16 - x^2}$ on the closed interval $[1, 3]$. Indicate at which $x$-values each of these extrema occur.

**Problem 12** Consider the function

$$y = \frac{5x}{x-1} + 1.$$

a) Find the equation of the tangent line to the graph of this function at $x = 6$.

b) Approximate $y$ for $x = 6.02$. Use the equation from part a) (amounts to the same as using differentials).

**Problem 13** A stock has price $p(t)$ at time $t$ and the demand for it is $x(t)$. Suppose $p$ and $x$ are differentiable and $p'(t) = 0.3$, $x'(t) = -0.025$ at a time when $p = 400$ and $x = 100$.

a) Find the rate of change of revenue at this time.

b) Is the revenue increasing or decreasing at this time?

**Problem 14** Evaluate the indefinite integral $\int \frac{3\,dx}{2x - 5}$. 
Problem 15  Evaluate the indefinite integral $\int x\sqrt{x^2 + 5} \, dx$.

Problem 16  Find the area enclosed by the graph of $f(x) = 20 - x^2$ and the line $g(x) = 7x + 12$.

Problem 17  Approximate the area between the $x$-axis, the graph of $f(x) = 1/\sqrt{x^3 + 1}$, and the lines $x = 1$ and $x = 3$ using four rectangles.
   a) Use the left endpoints of your subintervals of $[1, 3]$.
   b) Use the right endpoints of your subintervals of $[1, 3]$.