Math 151 - Exam 1A

Show all work in the space provided. Calculators are allowed, but no cellphones, books, notes. Give exact results, no rounded decimal fractions except in Problem 9. If a limit is undefined, write DNE for 'does not exist'. Time is 50 minutes. All the best!

1. Solve for $x$: \( \left( \frac{1}{2} \right)^{1-x} = 8 \)

2. Solve for $t$: \( \log_3(2t - 1) = 3 \)

3. Write as a single logarithm: \( \frac{1}{5} \log_3(x^2 + 3) - 2 \log_3(x - 1) \)

4. How many years will it take to double a principal of $1000 at 7\% interest, compounded continuously? Give the exact answer, and round to one digit after the decimal point.
5. \( \lim_{u \to -3} \frac{u^2 + u - 6}{u^2 + 5u + 6} \)

6. Consider the function \( f(x) = x^2 - 3x + 1 \).
   a) Find the average rate of change \( f_{1,3} \) for \( f \) between \( x = 1 \) and \( x = 3 \).
   
   b) Find the instantaneous rate of change of \( f(x) \) at \( x = 1 \).

7. Find \( \lim_{x \to 4^-} \frac{6 - x}{x - 4} \).
8. Let \( f(x) = \begin{cases} 
  x + 1 & \text{if } x < 0 \\
  x^2 - 2x & \text{if } 0 \leq x < 3 \\
  x & \text{if } x > 3 
\end{cases} \)

Find the following limits. Write DNE for a limit that does not exist.

a) \( \lim_{x \to 0^-} f(x) = \)

b) \( \lim_{x \to 0^+} f(x) = \)

c) \( \lim_{x \to 0} f(x) = \)

e) \( \lim_{x \to 3^-} f(x) = \)

f) \( \lim_{x \to 3^+} f(x) = \)

g) \( \lim_{x \to 3} f(x) = \)

h) Find all points \( x \) where \( f(x) \) is not continuous.
9. We model the population of a major city $P(t)$ at time $t$ with an exponential law,

$$P(t) = Ce^{kt}$$

a) The population in 1980 (at $t = 0$) was 5 million and is 6 million in 2012 (at $t = 32$). Find the values of the constants $C$ and $k$.

b) When will the population be 10 million if the population growth continues to follow this law? Round your answer to a whole number of years.

c) Approximate the rate of change of $P(t)$ in 2012 (at $t = 32$), using your calculator. Round the answer to four digits after the decimal point.