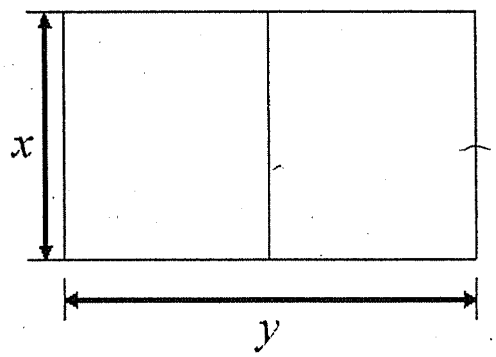
2. A farmer has 100 meters of wire fence with which he plans to build two identical adjacent pens, as shown in the figure below. What are the dimensions of the enclosure that has maximum area? (10 points)



Sol: The length is denoted by

$L = 3x + 2y = 100$ and the dimension is

$A = x \cdot y$

Thus: to maximize A. $\Leftrightarrow$

$$\max \quad A(x) = x \cdot \frac{100-3x}{2} = 50x - \frac{3}{2}x^2$$

and $x \in [0, 100]$. So:

The critical pts are: i) stationary pts: $A' = 0 \Rightarrow 50 - 3x = 0$

$$\Rightarrow x = \frac{50}{3}$$

ii) ending pts: $x = 0, 100$

iii) smoothness guarantees No singularities

So $A(0) = 0$, $A(100) < 0$ & , $A\left(\frac{50}{3}\right) = \frac{50^2}{3} - \frac{3}{2} \cdot \frac{(50)^2}{9}$

$$= \frac{50^2}{3} - \frac{(50)^2}{3 \cdot 2} = \frac{2500}{6}$$

note $A'' = -3 < 0 \Rightarrow$ concavity $\Rightarrow$ maximum

So. A is maximized to be $\boxed{\frac{2500}{6}}$ as $\boxed{x = \frac{50}{3}, \; y = 25}$