

Math 181
Name:
March 3, 2008

QUIZ # 6
(TAKE HOME. DUE ON MONDAY, MAR 3)

Find the derivatives of the following functions

1. (4 points) $y = \sin x \cos x$

Solution: Notice that $\sin(2x) = 2 \sin x \cos x$, by chain rule

$$y' = \left(\frac{1}{2} \sin 2x\right)' = \frac{1}{2} \cos(2x)2 = \cos 2x.$$

Or, by product rule

$$y' = (\sin x)' \cos x + (\cos x)' \sin x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = \cos 2x.$$

2. (4 points) $y = \sec^3 x$

Solution: By chain rule, let $f(x) = x^3$ and $g(x) = \sec x$ so that $y = f(g(x))$, we have

$$y' = f'g' = 3(g(x))^2(\sec x)' = 3 \sec^2 x \tan x \sec x = 3 \sec^3 x \tan x.$$

3. (4 points) $y = (x^3 + 1)^{10}$

Solution: Let $f(x) = x^{10}$ and $g(x) = x^3 + 1$, then $y = f(g(x))$. By chain one, we have

$$y' = f'g' = 10(x^3 + 1)^9(3x^2) = 30x^2(x^3 + 1)^9.$$

4. (4 points) $y = \frac{2}{\sin^3 x}$

Solution: Let $f(x) = \frac{2}{x^3}$ and $g(x) = \sin x$, so that $y = f(g(x))$. Using chain rule, we have

$$y' = f'g' = 2(-3)(g(x))^{-4} \cos x = -\frac{6 \cos x}{\sin^4 x}.$$

5. (4 points) $y = \sin^3(x^2 - 2x)$

Solution: Letting $f(x) = x^3$, $g(x) = \sin x$ and $h(x) = x^2 - 2x$, so $y = f(g(h(x)))$. By chain rule,

$$y' = f'g'h' = 3(g(h(x)))^2 \cos(h(x))(2x - 2) = 3(\sin^2(x^2 - 2x)) \cos(x^2 - 2x)(2x - 2).$$