

Math 181
Name:
February 8, 2008

QUIZ # 4

Find the following limits

1. (4 points) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 + 6x - 16}$

Solution: $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 + 6x - 16} = \lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{(x+8)(x-2)} = \frac{x+3}{x+8} \Big|_{x=2} = \frac{1}{2}$.

2. (4 points) $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x-4}$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x-4} &= \lim_{x \rightarrow 4} \frac{(\sqrt{2x+1} - 3)(\sqrt{2x+1} + 3)}{(x-4)(\sqrt{2x+1} - 3)} \\ &= \frac{2x-8}{(x-4)(\sqrt{2x+1} + 3)} \Big|_{x=4} = \frac{1}{3}. \end{aligned}$$

Find the derivatives of the following functions

3. (4 points) $f(x) = 2x^3 - 4x + 1$

Solution: $\frac{D}{Dx} f(x) = \frac{D}{Dx}(2x^3) - \frac{D}{Dx}(4x) + \frac{D}{Dx}(1) = 6x^2 - 4 + 0 = 6x^2 - 4$.

4. (4 points) $f(x) = 5x^3 - 17x^2 + 2\pi x + 3$

Solution: $\frac{D}{Dx} f(x) = \frac{D}{Dx}(5x^3) - \frac{D}{Dx}(17x^2) + \frac{D}{Dx}(2\pi x) + \frac{D}{Dx}(3) = 15x^2 - 34x + 2\pi$.

5. (4 points) $f(x) = \sqrt{2x+1} + \frac{2}{x} + x^2$

Solution: $\frac{D}{Dx} f(x) = \frac{D}{Dx}(\sqrt{2x+1}) + \frac{D}{Dx}\left(\frac{2}{x}\right) + \frac{D}{Dx}(x^2) = (2x+1)^{-1/2} - 2x^{-2} + 2x$.

Bonus point problem (5 points)

What is the derivative of the function

$$g(x) = \frac{1}{3x+4},$$

and what is the equation of the line tangent to $y = g(x)$ when $x = 2$?
 Solution:

$$\begin{aligned}
 \frac{D}{Dx}(g(x)) = m &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{3(x+h)+4} - \frac{1}{3x+4}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{3x+4 - (3x+3h+4)}{(3x+3h+4)(3x+4)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-3h}{(3x+3h+4)(3x+4)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-3}{(3x+3h+4)(3x+4)} \\
 &= \frac{-3}{(3x+4)^2}
 \end{aligned}$$

so that the slope at $x = 2$ is

$$m = g'(2) = -\frac{3}{100}.$$

By slope point formula, for $x = 2$ the corresponding $y = g(2) = 1/10$. Therefore the tangent line's equation is

$$y - \frac{1}{10} = -\frac{3}{100}(x - 2) \Rightarrow y = -\frac{3}{100}x + \frac{4}{25}.$$