

Math 181
Name:
February 1, 2008

QUIZ # 3

1. (5 points) Mold grows in a circular colony on a Petri dish containing an agar gel with the appropriate nutriment. The area A of the mold colony (in mm^2) is a quadratic function of the number t of hours since the start of the experiment, so that it has the form

$$A = at^2 + bt + c.$$

The measurement given in the experiments is as follow

t	A
0	1.35
18	5.30
48	23.85

- (i) What is the formula for the area of the colony in terms of time in hours.
(ii) Predict the area of the colony when $t = 60$ hours.
(iii) According to the formula you get, what is the time when the area is $80mm^2$?

Solution:

- (i) Plug the data back to the equation, one has linear system

$$\begin{cases} 0 + 0 + c = 1.35 \\ 18^2a + 18b + c = 5.3 \\ 48^2a + 48b + c = 23.85 \end{cases}$$

so that

$$a = 0.00831, \quad b = 0.0699, \quad c = 1.35.$$

Then the quadratic equation is

$$A(t) = 0.00831t^2 + 0.0699t + 1.35.$$

- (ii) When $t = 60$, according to the equation, one has

$$A(60) = 0.00831 \times 60^2 + 0.0699 \times 60 + 1.35 = 35.5mm^2.$$

- (iii) When $A = 80$, then the quadratic equation is

$$80 = 0.00831t^2 + 0.0699t + 1.35 \Rightarrow 0.00831t^2 + 0.0699t - 78.65 = 0.$$

Therefore, by the quadratic formula, one has

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.0699 \pm \sqrt{0.0699^2 + 4 \times 0.00831 \times 78.65}}{2 \times 0.00831}.$$

Disregarding the negative one, one has $t = 93.2$ hours.

t	0	0.5	1
H	0	5	2

2. (6 points) A man stands on the edge of a deep pit in the ground. He stoops and throws a stone upward, so that it first rises and then drops into the pit. The height H of the stone (in feet) above the ground at time t seconds after the rock is dropped is given by the data

The height is given by a quadratic equation of the form

$$H(t) = at^2 + bt + c.$$

- (i) What is the formula for the height of the rock in terms of time t .
(ii) Tell the height of the rock after 0.25 seconds.
(iii) Tell the time when the stone passes the ground level again.
(iv) The stone hits the bottom of the pit after 6.13 seconds. How deep is the pit?

Solution:

- (i) Plug the data back to the equation, one has linear system

$$\begin{cases} 0 + 0 + c = 0 \\ 0.25a + 0.5b + c = 5 \\ a + b + c = 2 \end{cases}$$

so that

$$a = -16, b = 18, c = 0.$$

Then the quadratic equation is

$$H(t) = -16t^2 + 18t.$$

- (ii) $H(0.25) = -16 \times (0.25)^2 + 18 \times 0.25 = 3.5$ feet.
(iii) Passing ground again means

$$0 = -16t^2 + 18t \Rightarrow t = 0, \text{ or } t = 9/8 = 1.125 \text{ sec.}$$

Disregard $t = 0$, which is the initial starting time, the answer is $t = 1.125$.

- (iv) $H(6.13) = -16 \times (6.13)^2 + 18 \times 6.13 = -490.9$, so that the depth of the pit is 490.9 feet.

3. (9 points) Mold growth experiment is done with the data given below. Based on the given data, find linear regression. (Recall: for linear regression, you suppose to have equation $y = a_0 + a_1x$. The matrix expression of the regression should be

$$\begin{pmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix},$$

where $y = a_0 + a_1x$.)

t	3	12	24	36	48	72	84
A	3.92	5.74	9.95	16.51	24.83	48.75	64.97

Solution: According the formula, you should have

$$n = 7, \sum x_i = 279, \sum x_i^2 = 16569, \sum y_i = 174.67, \sum x_i y_i = 11073.12.$$

Therefore, the system reads

$$\begin{pmatrix} 7 & 279 \\ 279 & 16569 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 174.67 \\ 11073.12 \end{pmatrix},$$

so

$$a_0 = -5.12, \quad a_1 = 0.754.$$

Then the regression line reads

$$y = 0.754x - 5.12.$$

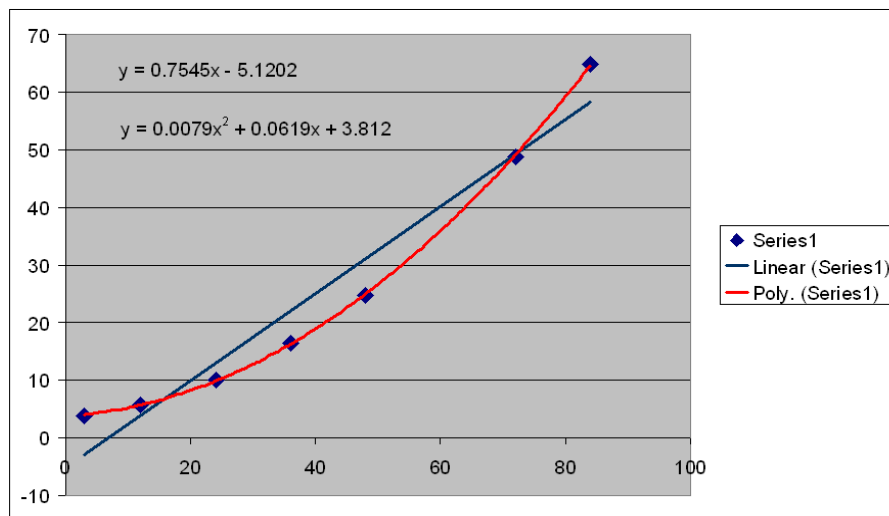


FIGURE 1