

You must do this project by yourselves, and you are not allowed to discuss with others about the problems. **The answer for each problem must be neatly and clearly written down.** Only solution will cause no points. **DUE ON MAY 14 IN CLASS.** (Total is 60 points)

1. (10 points) Suppose the particle potential changes with respect to time such that

$$\Phi(t) = \frac{3t^6 + 2t^2 - 5t + 4}{t^2}$$

- (a) What is the instantaneous rate of change of the potential of f when $t = 1$?
- (b) Find the tangent line to the potential function at $t = 2$?
- (c) Physically, the potential is the integral of the external and internal forces. So, the derivative of the potential is the rate of change of the total forces? What is the rate of change for the total forces at $t = 1$?

2. (10 points) Temperature is commonly measured in two different scales—the Fahrenheit scale with units $^{\circ}\text{F}$, and the Celsius scale with units $^{\circ}\text{C}$. Water freezes at 0°C , which in Fahrenheit is recorded as 32°F . Water boils at 100°C , while in Fahrenheit the boiling temperature is 212°F . If F is the temperature in $^{\circ}\text{F}$ and C is the temperature in $^{\circ}\text{C}$, then a straight line gives the relationship.

(a) Find the equation for F in terms of C .

(b) The usual body temperature of a human is 98.6°F . What is the usual body temperature measure in degrees Celsius?

(c) A recipe in a French cookbook tells you to bake at 200°C . Your American oven uses degrees Fahrenheit. At what temperature should you set your oven?

3. (10 points) A yield of corn is a function

$$Y = \frac{10F}{2 + F^2}$$

for $F \geq 0$ where F is the amount of fertilizer in the solid in tones/acre and Y is measure in tones/acre.

(a) Find the level of fertilizer that will maximize the yield.

(b) Find the maximum yield.

4. (5 points) A model for the concentration $x(t)$ of carbon dioxide in the blood at time t involves the quantity

$$V(t) = \frac{x(t)^{1.4}}{2 + x(t)^{0.4}}.$$

If $x(3) = 0.5$, and $x'(3) = 0.12$, what is $V'(3)$?

5. (15 points) Suppose that a population of fish contains p tons of fish at the end of the year. It is known that the following year the natural population will consist of $F(p)$ tons of fish, where

$$F(p) = 1.035p - 0.015 \ln(p).$$

- (a) What is the equilibrium population?
- (b) What is maximum sustainable harvest? And, what is the corresponding breeding population at the maximum sustainable harvest? (Hint: read section 4.4 carefully, and as you understand the concepts there, this problem is trivial.)

6. (10 points) A population of deer oscillates due to periodic predation. The population of deer t years after the base time is

$$p(t) = 24 + 11 \sin(0.898t - 0.211)$$

thousand deers.

- (a) What is population 3.4 years after the base time?
- (b) What is the minimum time that there is a maximal population of deer? And, what is the minimum time that there is a minimum population of deer?
- (c) What is the maximum number of deer in the population?
- (d) Tell the first two successive positive times t when the number of deer is maximized?