Measuring the Dispersion Coefficient with Acoustic Doppler Current Profilers

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Abstract: A method is evaluated for estimating the longitudinal dispersion coefficient K from velocities and bathymetry measured with an acoustic Doppler current profiler (ADCP). If shear dispersion controls the mixing, the dispersion coefficient can be estimated from measurements of velocity and depth in a cross section. The dispersion coefficient has typically been measured by costly and time-consuming tracer studies because the velocity field could not be resolved sufficiently before the flow changed. However, ADCP transects, which now are routinely used to measure discharge, provide detailed velocity and bathymetry data quickly. The dispersion coefficient is estimated from ADCP measurements from the United States Geological Survey and compared with estimates from dye studies. Half of the estimates of K fall within 50% of the values from tracer studies, and 85% are within a factor of 3. The ADCP method is at least as accurate as the best empirical formula considered. Both the comparison of field data and an analysis with theoretical velocity profiles suggest that the error in K will be largest when the velocity profile is nearly uniform.

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Introduction

Predicting the spread of contaminants is important for managing and protecting rivers and streams. To simulate contaminant dispersion, most mixing models require a longitudinal dispersion coefficient, which depends on the river geometry and flow. The dispersion coefficient has generally been estimated with empirical formulas or costly field tracer experiments. However, empirical formulas estimate K only within an order of magnitude (Rutherford 1994). Tracer studies are considered more accurate, but the results apply for only the reach examined and the flow and weather during the study. Tracer studies also require a large investment in planning, staff, and analysis.

Another approach to determining the longitudinal dispersion coefficient is to estimate it directly from the theory of shear dispersion with (Fischer 1967)

$$K = -\frac{1}{A}\int_0^B u'(y)h(y)dy\int_0^y \frac{1}{D_y h(y')}dy'\int_0^{y'} u'(y'')h(y'')dy'' \tag{1}$$

where y=transverse coordinate that runs from 0 at one bank to the width B at the other; A=cross-sectional area; h=depth; D_y=transverse mixing coefficient; $u'(y)=u(y)-\overline{U}$=velocity deviation; $u(y)$=depth-averaged streamwise velocity; and $\overline{U}$=velocity averaged over the cross section. This equation assumes that shear dispersion, the mixing caused by particles experiencing different velocities as they are randomly diffused back and forth across the river by transverse dispersion, is the main mixing mechanism in the longitudinal direction.

Use of Eq. (1) is limited by the assumptions in the theory. In particular, the flow is assumed to be one-dimensional; that is, the contaminant must be well mixed in the transverse and vertical directions. This assumption may limit the validity of Eq. (1) in regions where the contaminant is not well mixed; in recirculation zones, which prevent the decay of the concentration profile to a Gaussian profile; and at bends, where strong secondary currents are present (Fischer 1969). Eq. (1) also requires the width of the river to be much larger than the depth, so that the transverse shear, and not the vertical shear, controls the dispersion. Further, the transverse velocity gradient must be large enough for shear dispersion to dominate over other spreading mechanisms. This assumption may fail in slowly moving reaches of a river, such as those with recirculation zones, or in rivers where other mechanisms may be important.

Use of Eq. (1) is also limited by practical considerations. Measuring detailed velocity and bathymetry in a cross section has been difficult; velocity has been measured by propeller-type or other conventional meters, which are limited to small streams where enough measurements can be made before the flow changes (Fischer et al. 1979). However, velocity and bathymetry are now measured with greater speed and detail with acoustic Doppler current profilers (ADCP), which obtain velocity from the Doppler shift of sound reflected from particles in the flow. ADCPs have been widely used in river measurements of discharge (e.g., Yorke and Oberg 2002); when mounted on a boat, ADCPs yield detailed snapshots of the vertical profiles of velocity at many transverse locations. Compared with traditional methods of measuring velocity, ADCPs provide much better spatial resolution and less disturbance of the flow. Evaluating the dispersion coefficient

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from such data will require less time and fewer people than a conventional tracer study. Bogle (1997) addressed further practical challenges with the ADCP method, including dealing with the ADCP's inability to measure near the banks and surface and estimating the transverse mixing coefficient. Several empirical estimates of the transverse mixing coefficient have been used, which have been derived from laboratory experiments (Elder 1959), theoretical velocity distributions (Fischer 1969), and, more recently, regression of field data (Deng et al. 2001).

To determine whether ADCP measurements will give a good estimate of the dispersion coefficient, we compare K from the ADCP method to values from tracer studies in several rivers. The dispersion coefficient is computed from ADCP measurements made by the U.S. Geological Survey (USGS) at gaging stations at ten U.S. rivers and compared with historical tracer study data measured at or near the site in the past 50 years as well as estimates from empirical formulas. Sources of error in the ADCP method are discussed to help identify rivers for which an ADCP measurement of the dispersion coefficient approaches the value from tracer studies. In particular, artificial velocity and bathymetry profiles are used to determine the error caused by missing velocity measurements near banks.

Methods

Comparison of Dispersion Coefficients

The dispersion coefficient was computed from ADCP measurements taken from 2000 to 2004 on ten rivers and compared to K determined from tracer studies conducted at approximately the same site at similar flows from 1967 to 1991 (Table 1). When the studies reported time series of tracer concentrations at more than one site, the routing method (Rutherford 1994) was used to determine the dispersion coefficient. In this approach, dispersion of a dye cloud as it passes a measuring site is neglected, and an upstream profile is routed downstream with a solution to the advection–dispersion equation; the value of K is varied until the routed profile matches the measured downstream profile. If the studies only reported data at a single site or specific cloud properties such as times and concentrations of the peak, leading edge, and trailing edge, the tracer response curve was modeled as a scalene triangle (Jobson 1997) and K was estimated by fitting a line to the variance, which is computed by integrating the concentration profiles, as a function of time (Fischer et al. 1979, p. 132).

The ADCPs used in the measurements reviewed here were downward looking, RD Instruments Rio Grande ADCPs with frequencies of either 600 or 1,200 kHz. They had four beams arranged with right angles between adjacent beams. The ADCP measures the velocity of the water in the direction of the beam at several points along the beam. Velocities in the coordinate directions were computed by assuming that the mean velocities in each beam were equal. The ADCPs were mounted to boats or towed on lines across the rivers to provide detailed measurements of vertical profiles of velocity as well as the bottom depths at many points in the cross section.

Boat speed, water speed, and depth were used to choose the bin sizes (or the vertical resolution of the velocity profile). Boat speeds ranged from 0.1 to 0.6 m/s, and the resulting times to measure a transect were between 1 and 14 min. Bin sizes ranged from 5 to 50 cm. Therefore, the ADCPs provided many more velocity measurements than propeller meters traditionally have provided. Location, water velocity, and boat speed were determined with either bottom tracking or a differential global positioning system (GPS). Bottom tracking can be more precise than GPS if the sediments on the bottom are stationary. When long time series at a fixed location indicated that the bottom was moving, GPS tracking was used, though accuracy worsens near banks because of overhanging trees. At least four transects were taken for each measurement, and more were measured if discharges were not within 5% of the mean. Measurements occurred at USGS gaging stations.

Processing the velocity profiles included removing spurious data, orienting the velocities, and estimating velocities to complete the profiles. Outlier depths were approximated from a mean of the surrounding data points. Velocities were removed in the bottom 6% of the water column, where sidelobe interference affects the measurements, and near the water surface, where acoustic ringing interferes with the signal. The velocities were then rotated into the streamwise direction by using the average flow direction of the set of transects.

To complete the profiles, velocities and bathymetry were estimated in unmeasured regions of flow and areas in which spurious data were removed. Bathymetry was approximated using a power law fit of the data points closest to the shore; this fit, which is similar to the channel bathymetry approximation in Deng et al. (2001) discussed in the following subsection, allowed for triangular, natural, and rectangular cross sections. Velocities were extrapolated using methods similar to those the USGS uses in discharge measurements. Velocities at the surface were extrapolated with a constant, a linear fit of several near-surface velocities, or a power fit over the entire depth. Bottom velocities were extrapolated from either the power fit or a linear fit from the deepest velocity measurement to zero at the bed. The power fit is based on a formulation for flow resistance (Chen 1992). Extrapolations for the unmeasured near-bank regions were determined from transverse velocity profiles presented by Bogle (1997), Deng et al. (2001), and Seo and Baek (2004). When none of the theoretical profiles fit well, power fits such as those used for the bathymetry were used. The bottom depths and velocities of the set of transects were then aligned to provide the closest fit in the transverse direction using either the correlation between depths of profiles or GPS data. An average depth profile and an average velocity profile for a set of transects were computed to reduce the fluctuations of the instantaneous measurements.

Once the velocity profiles were complete, the transverse mixing coefficient was approximated from the relation $D_y = \theta u_* H$ (Rutherford 1994), where u_* = shear velocity and the coefficient θ is calculated as (Deng et al. 2001)

$$\theta = 0.145 + \frac{1}{3{,}520}\left(\frac{\overline{U}}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} \tag{2}$$

Several methods were available to estimate u_*. The most appropriate reach scale estimate was $u_* = (gRS_0)^{1/2}$, where g = acceleration of gravity; R = hydraulic radius; and S_0 = channel slope; this method was used when good slope estimates were available and uniform flow was a reasonable assumption. If long time series at a fixed position were collected, the shear velocity was determined from the slope of the logarithmic law (e.g., Kundu and Cohen 2004, p. 554) fit to the ensemble average velocity profile. Shear velocity was also estimated from a theoretical transverse velocity profile (Sooky 1969) and the logarithmic law for a fully turbulent flow integrated over the cross section. Once the above approximations were made, K was calculated for each

Table 1. Rivers, Properties, and Dispersion Coefficients Calculated from Both the Acoustic Doppler Current Profiler (ADCP) Measurements of the U.S. Geological Survey (2005) and Tracer Studies

River and site*	B (m)	H (m)	$\overline{U}$ (m/s)	u_* (m/s)	Q (m^3/s)	K (m^2/s) Tracer	ADCP
Big Blue River[a]							
Marysville, Kan.	75	1.6	0.22	0.990† [i]	26.7	17.0	11.8
Embarrass River[b]							
St. Marie, Ill.	30	1.1	0.38	0.025†	13.9	35.9	8.0
Illinois River[c]							
Henry, Ill.	158†	4.3†	0.19	0.007†	66.8	48.9	41.1
Henry, Ill.	232†	3.4†	0.24	0.043†	234.4	52.0	223.8
Kingston, Ill.	202	4.6	0.18	0.036†	162.3	49.1	71.7
Kingston, Ill.	194	6.3	0.22	0.039†	272.7	537.7	474.6
Marseilles, Ill.	183	5.7	0.11	0.020†	111.1	13.3	10.9
Kanawha River[d]							
Charleston, W.Va.	259	3.3	0.17	0.017†	141.6	24.2	50.9
Charleston, W.Va.	259	3.4	0.17	0.018†	147.3	22.1	51.7
Missouri River[e]							
Decatur, Neb.	230	3.5	1.08	0.085	883.5	455.1	155.9
Omaha, Neb.	176	3.4	1.61	0.082	945.8	966.2	64.9
Sioux City, Iowa	229	3.4	1.24	0.082	948.6	309.8	123.0
New River[f]							
Hinton, W.Va.	102	4.4	0.17	0.008†	93.8	22.4	50.5
Ohio River[g]							
Sewickley, Pa.	388	6.2	0.05	—	130.1	6.3	17.0
Salt Creek[a]							
Pioneers, Neb.	167	0.2	0.47	0.159†	14.4	43.2	24.4
Sangamon River[b]							
Monticello, Ill.	27	1.1	0.44	0.007†	12.8	24.6	10.3
Yampa River[h]							
Below Craig, Colo.	78	1.2	1.42	0.026†	130.5	325.6	290.4
Deerlodge, Colo.	300	0.3	1.00	0.029†	79.3	349.6	271.1
Deerlodge, Colo.	300	0.4	0.97	0.032†	127.1	227.7	183.4
Near Hayden, Colo.	76	1.2	1.41	0.058†	126.6	116.4	115.7

Note: Unless otherwise noted, values of river properties come from the tracer studies.

*Data with asterisk indicates sources for tracer studies:

[a]Petri (1984).

[b]Graf (1986).

[c]Zuehls (1987).

[d]Wiley (1993).

[e]Yotsukura et al. (1970).

[f]Appel and Moles (1987).

[g]Wiley (1997).

[h]Ruddy and Britton (1989) and Bauer et al. (1979).

[i]Values marked with a dagger were not reported in the tracer study and have been estimated from ADCP transect data.

measurement by evaluating the integrals in Eq. (1) numerically. Since the tracer studies and ADCP measurements occurred at different flows, dispersion coefficients were linearly interpolated between flows.

Effect of Unmeasured Data

The effect of unmeasured data near banks was evaluated by determining K from constructed theoretical velocity and bathymetry profiles and comparing with profiles from which data near banks had been removed. That is, if 20% of the velocity profile wasremoved, integrals in Eq. (1) were computed from $y=0.1B$ to $y=0.9B$. This comparison provides an upper bound on the error because in processing real velocity profiles, velocities in unmeasured regions are estimated, as in the previous subsection. For this analysis, a set of normalized velocity profiles was produced from a velocity distribution presented by Seo and Baek (2004)

$$\frac{u(y)}{\overline{U}} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{y}{B}\right)^{\alpha-1}\left(1-\frac{y}{B}\right)^{\beta-1} \tag{3}$$

The shape parameters α and β allow profiles with different skew and kurtosis to be represented. When $\alpha=\beta$, skewness is zero, whereas if $\alpha<\beta$, the profile is skewed to the left. As the differ-

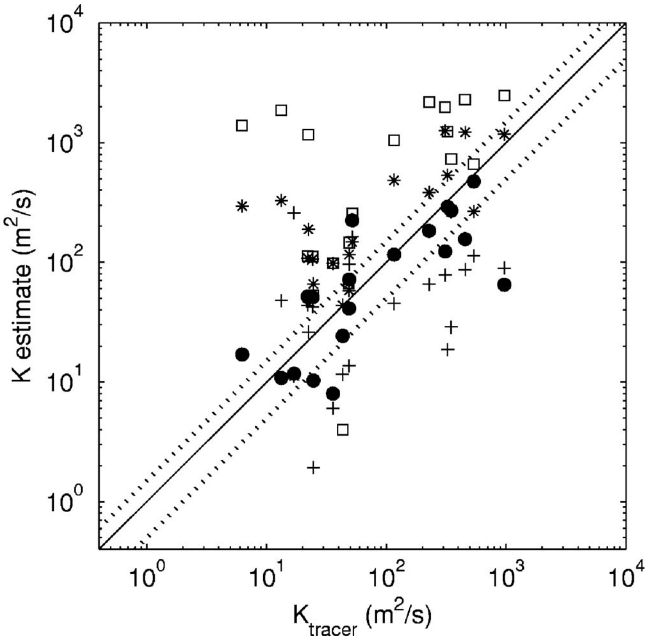

Fig. 1. Comparison of the dispersion coefficient estimated with the ADCP method (circle) and values of K determined from tracer studies. Also shown are estimates from empirical formulas: square, Fischer (1975); plus sign, Iwasa and Aya (1991); and asterisk, Seo and Cheong (1998). The solid line indicates equality and the dotted lines indicate 50% error.

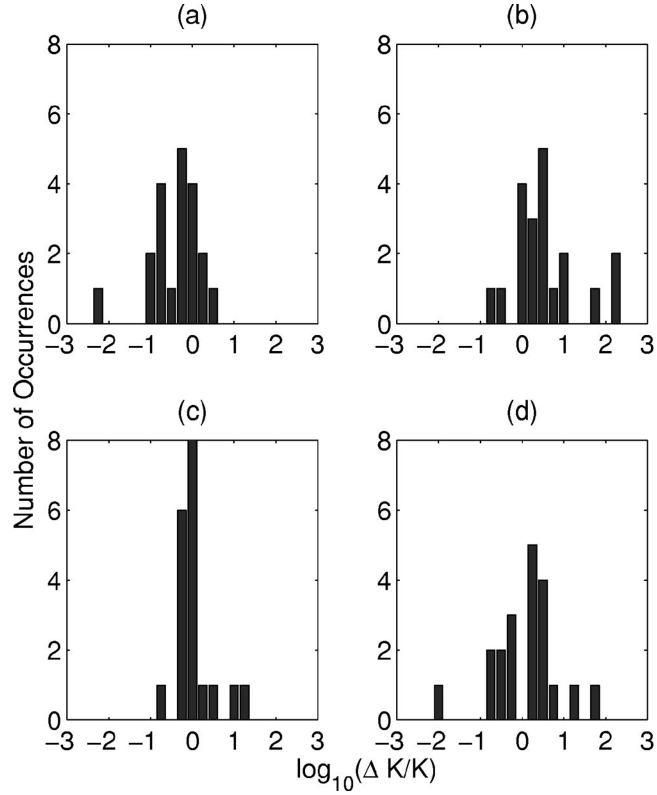

Fig. 2. Error in various estimates of the dispersion coefficient: (a) ADCP method; (b) Fischer (1975); (c) Iwasa and Aya (1991); and (d) Seo and Cheong (1998). A value of $\log_{10}(\Delta K/K)$ of zero corresponds to 100% error.

ence between α and β increases, the skewness increases. For zero skewness, the profile is sharper, or has a larger kurtosis, as α and β both increase above unity. Depth profiles were computed with (Deng et al. 2001)

$$\frac{h(y)}{H} = 1 - \left| \frac{2y}{B} - 1 \right|^{\gamma} \tag{4}$$

Velocity profiles of different skewness and kurtosis were considered for three bed shapes: triangular ($\gamma=1$), parabolic ($\gamma=2$), and natural ($\gamma=5$). Only the natural bed shape ($\gamma=5$) is discussed in the next section.

Results and Discussion

The ADCP method shows promise for estimating the dispersion coefficient. Of the 20 estimates of K, 10 have less than 50% error, and many are quite close to the values from the tracer studies (Fig. 1). Seven of the remaining estimates are within a factor of 3 of the tracer values. Trends are difficult to detect with the relatively low number of estimates, but when errors exceed 50%, the ADCP method tends to yield a low estimate for rivers with large longitudinal dispersion, and it produces both high and low estimates for rivers with smaller longitudinal dispersion. Overall, however, the method performs quite well.

The ADCP method performs as well as or better than several empirical formulas for many of the rivers (Fig. 2). Fischer's (1975) estimate mostly overpredicts K, as Seo and Cheong (1998) found. Seo and Cheong (1998) also concluded that the formula of Iwasa and Aya (1991) works relatively well; for these data, its estimates are consistently low for tracer values of K greater than 70 m^2/s. The formula of Seo and Cheong (1998) mostly overestimates K, but for rivers with larger dispersion, it reproduces the values of K from the tracer studies more closely.

Differences between the estimates of K with the ADCP method and the values from tracer studies could come from several sources. If the river geometry changes in the time between the tracer study and the ADCP measurements, the dispersion coefficient should also change. Therefore, our future work includes simultaneous tracer studies and ADCP measurements. Also, the ADCP method should be less accurate when the assumptions behind Eq. (1) are violated, for example, when the rivers have sharp bends that cause strong secondary flows, large recirculation zones that retain tracers and re-release them slowly, and small widths that lead to significant contributions of vertical gradients of velocity to dispersion. Because all rivers in this study had $B/H > 6$, they meet the criterion required for transverse gradients to dominate (Fischer 1967). The limited number of data points did not allow relationships between characteristics of the rivers, such as aspect ratio and Reynolds number, and the accuracy of the ADCP method to be determined. However, a plausible explanation for the underestimate of K at high tracer values is that mechanisms other than shear dispersion contribute to overall dispersion.

Because K estimated from Eq. (1) is inversely proportional to the transverse mixing coefficient D_y, uncertainty in D_y translates to uncertainty in K. Transverse mixing consists of both mixing by turbulent eddies and dispersion by secondary flows. For straight channels $\theta = D_y/u_* H$ is between 0.15 and 0.30, and as the channel curvature increases, θ can reach values up to 3 and, in rare cases, higher (Rutherford 1994, pp. 108–113). Values of θ computed with Eq. (2) and river parameters measured during the ADCP measurements, which do not necessarily match those during the

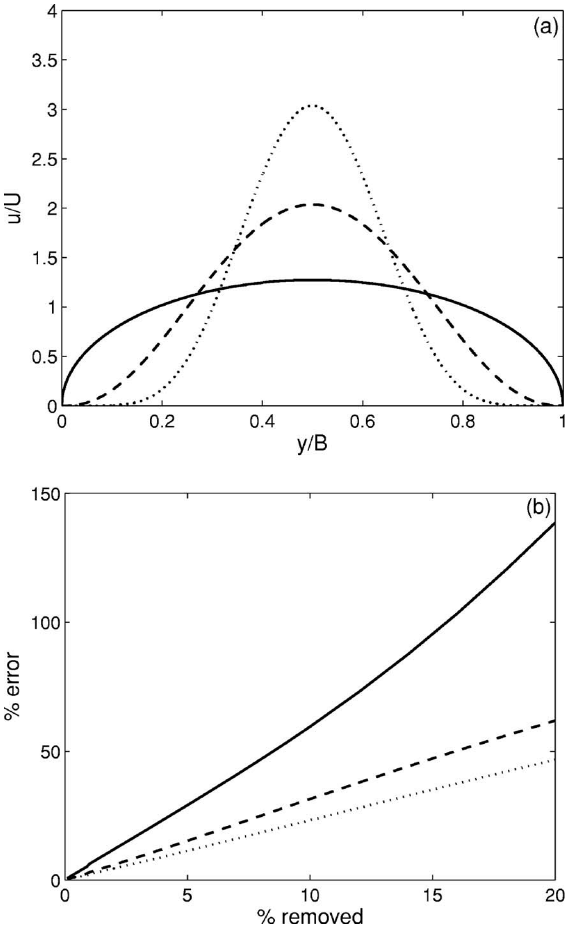

Fig. 3. Effect of missing data near banks: (a) sample theoretical velocity profiles with sharp peak, $\alpha = \beta = 7.5$ (dotted line), moderate peak, $\alpha = \beta = 3.5$ (dashed line), and mild peak, $\alpha = \beta = 1.5$ (solid line); (b) percentage error $(K_{ref} - K_{est})/K_{ref}$ as a function of the percentage of data removed from the edges of the theoretical profiles

tracer studies, vary between 0.15 and 2.8, as in the compilation of Rutherford (1994). Nevertheless, further tests to improve the estimates of the transverse mixing coefficient would improve the accuracy of dispersion coefficients estimated with the ADCP method.

Also, although dye studies provide values of the dispersion coefficient averaged over the reach, K estimated with Eq. (1) uses ADCP measurements at one cross section. Therefore, variations within the reach can cause the estimates from the two methods to differ. We recommend using several cross sections away from features that can cause irregularities (e.g., wakes) in the velocity profile and averaging the estimates of K from the transects; when advection dominates dispersion (i.e., large Péclet number), a weighted average can be used with the lengths of the subreaches as the weighting factors (e.g., Rutherford 1994, p. 217).

Discrepancy also comes from estimating the velocities in the parts of the cross section where the ADCP cannot measure. The effect of unmeasured data near banks was evaluated by comparing K computed from theoretical velocity and bathymetry profiles with and without near-bank data. Three typical normalized velocity profiles created from a velocity distribution presented by Seo and Baek (2004) are shown in Fig. 3(a). Data were removed equally from each side of the theoretical velocity profiles, and K computed from the partial profile was compared to K computed from the full profile [Fig. 3(b)]. Missing data lead to a low estimate because the profile appears more uniform. The error increases as more edge data are removed, with errors between 45 and 140% for 20% of the data removed. Error is largest for the profile with the mildest peak, or the most uniform velocity profile. Again, when the profile is more uniform, most of the shear dispersion is due to the velocity gradients near the edges of the profile.

The large error estimated in this analysis, in which near-shore data were completely removed, is an upper bound of the error caused by missing data. In practice, we recommend reducing this error by approximating the near-shore data with the methods used in analyzing the measurements summarized in Table 1. Uncertainty from these approximations can be reduced further by measuring near-shore velocities with other instruments, such as an acoustic Doppler velocimeter (ADV). However, if USGS measurements at gaging stations are to be used to estimate K, near-shore ADV measurements are unlikely to be available: Although the near-shore regions can affect the dispersion coefficient strongly, they contribute less to the discharge than other parts of the flow. As ADCP technology improves for measuring in shallow water, the need to estimate velocities in unmeasured regions or measure them with other instruments will decrease.

The effect of the profile shape helps to explain the larger errors observed in comparing the field measurements. Though often the measured velocity profiles were more skewed than the sample profiles in Fig. 3, the field results were classified by the shape parameter, $\alpha = \beta$. For the profiles with the mildest peaks ($\alpha < 2$), the average error was 99%, whereas for the profiles with sharper peaks ($2 < \alpha < 6$), the average error was 52%. For the profiles with the sharpest peaks ($\alpha > 6$), the average error was 31%. Therefore, the method may be more difficult to apply in rivers where most of the velocity gradient is near shores where ADCPs cannot measure. Nevertheless, missing data cannot explain all of the error in the comparison of field measurements, which exceeds the upper bound estimates from this analysis. Additional error must come from other sources, such as those discussed earlier.

Summary

Using ADCP velocity and bathymetry measurements to determine the longitudinal dispersion coefficient can reduce the effort and expense of measuring K in many rivers, increase the understanding of river mixing, and improve the accuracy of predictions of contaminant transport. Further, because the U.S. Geological Survey regularly measures river discharge with ADCPs, the dispersion coefficient can be estimated in many more rivers. We tested the ADCP method with measurements from ten rivers by comparing estimates of K with estimates from historical dye studies. The results are encouraging: Half of the estimates of K fall within 50% of the values from tracer studies, and 85% are within a factor of 3. The ADCP method performs better than two empirical formulas for the dispersion coefficient, and it is at least as accurate as the best formula considered. Error in the method's estimates come from several sources, including applicability of the shear dispersion theory to specific river conditions and limitations of ADCP measurements. An analysis with theoretical velocity profiles shows that the effect of missing data in parts of the profile where the ADCP cannot measure is largest for the most uniform velocity profiles. The explanation is consistent with observations from the comparison of field measurements.

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Notation

The following symbols are used in this technical note:

A = cross-sectional area;
B = top width;
D_y = transverse mixing coefficient;
g = acceleration of gravity;
H = cross-sectional average depth;
h = depth;
K = longitudinal dispersion coefficient;
Q = flow rate;
R = hydraulic radius;
S_0 = bed slope;
$\overline{U}$ = cross-sectional average velocity;
$u(y)$ = depth averaged streamwise velocity;
$u'(y)$ = deviation velocity;
u_* = shear velocity;
y = transverse coordinate;
α, β = shape parameters for velocity profile (Seo and Baek 2004);
Γ = gamma function;
γ = shape parameter for depth profile (Deng et al. 2001); and
θ = coefficient in approximation of D_y.

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