*10. In three dimensions, the $y$-momentum equation, where $y$ points up, is

$$ \frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial x} (\rho uv) + \frac{\partial}{\partial y} (\rho vv) + \frac{\partial}{\partial z} (\rho wv) = -\rho g + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}. $$

Show that for all flows the $y$-momentum equation can be written as

$$ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\rho g + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}. $$

11. Show that the $x$-momentum equation in three dimensions is

$$ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}. $$

*12. Edwards et al. (2005, Can. J. Fish. Aquat. Sci.) solved a one-dimensional model for the transport of phytoplankton. The model included sinking with a velocity $w_s$, net growth with a rate $\mu$, and turbulent flux $q_F$ (say). We will see later that turbulent fluxes are often modeled with a version of Fick’s law:

$$ q_F = -K \frac{\partial F}{\partial z}, $$

where $K$ is called the vertical eddy diffusivity. Find this paper and derive equation (1).