Constitutive relations in other coordinate systems

The handout from the book by Kundu and Cohen gives the equations for conservation of mass and momentum (which for Newtonian fluids such as air and water are called the Navier-Stokes equations) in cylindrical polar coordinates, plane polar coordinates, and spherical polar coordinates. That handout also contains information needed to compute the viscous stresses. The authors named the viscous stresses $\sigma_{ij}$, where the subscripts indicate the direction of the stress and the normal to the plane on which the stress acts. Adding the pressure to the normal viscous stress gives the full stress tensor.

For example, for cylindrical polar coordinates, the full stress tensor is

$$
\begin{align*}
\tau_{RR} &= -p + 2\mu \frac{\partial u_R}{\partial R} \\
\tau_{\theta\theta} &= -p + 2\mu \left( \frac{1}{R} \frac{\partial u_\theta}{\partial \theta} + \frac{u_R}{R} \right) \\
\tau_{xx} &= 2\mu \frac{\partial u_x}{\partial x} \\
\tau_{R\theta} &= \mu \left( R \frac{\partial}{\partial R} \left( \frac{u_\theta}{R} \right) + \frac{1}{R} \frac{\partial u_R}{\partial \theta} \right) \\
\tau_{\theta x} &= \mu \left( \frac{1}{R} \frac{\partial u_x}{\partial \theta} + \frac{\partial u_\theta}{\partial x} \right) \\
\tau_{xR} &= \mu \left( \frac{\partial u_R}{\partial x} + \frac{\partial u_x}{\partial R} \right)
\end{align*}
$$