Lec. 32: 4-3-09

1. Analogy

2. River mixing examples

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<td>[ C = \frac{u}{2} \frac{\partial C}{\partial y} ]</td>
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<td>[ C \text{ = mass flux} = \frac{u}{2} \frac{\partial C}{\partial y} ]</td>
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Unsteady flow to a well

\[ \frac{\partial s}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial s}{\partial r} \right) = \frac{1}{1 - \left( \frac{r}{R} \right)^2} \]

\[ s = \text{drawdown} \]
\[ r = \text{radial distance} \]
\[ t = \text{time} \]

\[ s = \frac{Q_0}{4\pi T} W(u) \]

\[ W(u) = \text{well function} \]

\[ u = \frac{Q_0}{4\pi T} \left( \frac{r}{2 \sqrt{D \tau}} \right) \]

\[ q = -\frac{\partial C}{\partial r} \]

**Darcey's law**

\[ \text{hydraulic conductivity} \]
River mixing

Instantaneous, dumping - spill
dye study

Contaminant mixed across river

Top view

First consider negligible velocity (i.e. tank)

- Peak remains at x=0
- Cloud spreads
- Peak concentration decreases log cons. of mass
- Damping

Cloud width δ ~ \sqrt{t}

Solution: \[ C = \frac{M}{A \sqrt{4\pi t}} \exp\left(\frac{-x^2}{4t}\right) \]

Gaussian

Add flow or mean velocity \( U \)
Just move with the flow:

\[ x \rightarrow x-Ut \]

\[ C = \frac{M}{A \sqrt{4\pi t}} \exp\left(\frac{-(x-Ut)^2}{4t}\right) \]