Lec. 29: 3-27-09

1. Unsteady viscous flow
2. Connections

Review

1. Framework
   - Cons. of mass
   - Cons. of momentum

2. Slot flow
   - Viscous slot flow: pressure ~ viscous
   - Extensions
     - Particle settling
     - Groundwater
     - Open channel flow & turbulence
Unsteady viscous flow (§9.7)

No pressure gradient
Long slot
Wall moves at speed \( U_0 \) starting
at time \( t = 0 \)
Quiescent at \( t = 0 \) (initial condition)

Known
1. BC: \( u = 0 \) at \( y = H \), \( u = U_0 \) at \( y = 0 \)
2. IC: \( u = 0 \) at \( t = 0 \)

Dimensional analysis
\[ u = f(H, v, k_3, B, U_0, t, x, y) \]

Simplify by restricting the problem
1. Two-dimensional \( \Rightarrow \) Ignore \( z \) \& \( B \)
2. Smooth walls \( \Rightarrow \) \( k_3 \ll H \)
3. Fully-developed or uniform \( \Rightarrow \) Neglect changes in \( x \)
4. \( U_0 \) merely sets scale of velocity
\[
\frac{u}{U_0} = f\left( \frac{y}{H}, \frac{v}{U_0}, \frac{t}{H^2} \right) = f\left( \frac{y}{H}, \frac{t}{H^2} \right)
\]
5. What if \( H \to \infty \)?
x-momentum

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]

Consider short time or large H

compare terms

\[
\frac{\nu \partial^2 u}{\partial x^2} \sim \frac{\nu U_0 / L^2}{\nu U_0 / L^2} = \left( \frac{\delta}{L} \right)^2 \ll 1
\]

\( \rightarrow \) If ratio is small, neglect x-viscous

or if \( H/L \ll 1 \rightarrow \) OK too.

\[
\frac{u \partial^2 u}{\partial y^2} \sim \frac{U_0 \partial U_0}{\nu \partial \delta / \delta^2} = \frac{U_0 \delta}{\nu L}
\]

Balance unsteady inertia and y-viscous

\[
\frac{\partial^2 u}{\partial t} \sim \nu \frac{\partial^2 u}{\partial y^2} \Rightarrow \frac{U_0}{t} \sim \nu \frac{U_0}{\delta^2} \Rightarrow \delta \sim \sqrt{Ut}
\]

Reconsider \( u \partial^2 u \). To neglect it, need

\[
\frac{U_0 \delta}{\nu L} = \frac{U_0 (\sqrt{Ut})}{\nu L} = \frac{U_0 t}{L} \ll 1 \Rightarrow L \gg U_0 t
\]