*27. Sketch profiles of the water surface as it moves between two lakes over a long, steep slope.

a. The level of the downstream lake is below the normal depth. What must the depth be at the start of the slope?

b. The level of the downstream lake is between the normal depth and critical depth. What must the depth be at the start of the slope?

c. The level of the downstream lake is above the critical depth. (Hint: This case itself will have two sub-cases.)
28. A rectangular channel excavated in rock \((n = 0.034)\) has a slope of \(4 \times 10^{-3}\) and a width of 10 m, and it carries a discharge of 20 \(m^3/s\). After a long distance, the channel is lined with concrete \((n = 0.01)\) for a section of 300 m, and then the channel returns to rock for a long distance. Compute the water surface profile and plot the depth as a function of distance. You may compute the length \(L\) of the hydraulic jump with

\[
\frac{L}{h_1} = 220 \tanh \frac{Fr_1 - 1}{22},
\]

where \(h_1\) and \(Fr_1\) are the depth and Froude number upstream of the jump.

29. Microbes in a batch reactor will grow and decay through death and respiration, which does not release organic carbon. Growth depends on the amount of substrate available. A model of microbial kinetics can be written as

\[
\begin{align*}
\frac{dX}{dt} &= \left(k_{g,max} \frac{S}{k_s + S} - k_d - k_r\right) X \\
\frac{dS}{dt} &= -\frac{1}{Y} k_{g,max} \frac{S}{k_s + S} X + k_d X
\end{align*}
\]

where \(X\) is the concentration of bacteria and \(S\) is the concentration of the substrate. The death rate is \(k_d\), and the respiration rate is \(k_r\). Growth is modeled with Michaelis-Menten kinetics such that when the substrate concentration is much greater than the half-saturation coefficient \(k_s\), the growth rate is \(k_{g,max}\); when \(S\) is smaller, the growth rate declines. Thus, the first equation states that the concentration of bacteria will increase because of growth and decrease because of death and respiration. The second equation states that substrate will be consumed by bacteria growth (including a cell yield \(Y\)) and created by the death of cells.

Solve the system numerically with the forward Euler approach and plot the concentrations of bacteria and substrate for a 50-hour period. Take \(k_{g,max} = 0.2 \text{ h}^{-1}\), \(k_d = k_r = 0.01 \text{ h}^{-1}\), \(k_s = 100 \text{ mgC/L}\), and \(Y = 0.5 \text{ gC cells/gC substrate}\). Take the initial concentrations of the bacteria and substrate to be 2 mgC/L and 998 mgC/L, respectively.
Ammonia from either direct loading or the decomposition of organic nitrogen is oxidized to form nitrite and nitrate, and this process consumes oxygen. Assuming first-order reactions, we can model nitrification with the following set of differential equations:

\[ \frac{dN_o}{dt} = -k_{oa} N_o \]
\[ \frac{dN_a}{dt} = k_{oa} N_o - k_{ai} N_a \]
\[ \frac{dN_i}{dt} = k_{ai} N_a - k_{in} N_i \]
\[ \frac{dN_n}{dt} = k_{in} N_i \]
\[ \frac{dD}{dt} = r_{oa} k_{ai} N_a + r_{oi} k_{in} N_i - k_a D \]

Here \( N_o, N_a, N_i, \) and \( N_n \) are the concentrations of organic nitrogen, ammonium, nitrite, and nitrate, respectively. The first equation says that organic nitrogen decomposes by a first-order reaction with rate coefficient \( k_{oa} \). The second says that ammonium increases by the decomposition of organic nitrogen and decreases by the oxidation to nitrite with rate coefficient \( k_{ai} \). The third equation is similar to the second: Nitrite increases because of oxidation of ammonium and decreases by oxidation to nitrate with a rate coefficient \( k_{in} \). Nitrate increases because of oxidation of nitrite. Then the deficit \( D = DO_{sat} - DO \) of dissolved oxygen \( DO \) from the value at saturation \( DO_{sat} \) increases because of the oxidation of ammonium and nitrate and decreases because of reaeration with a rate coefficient \( k_a \). The ratios \( r_{oa} \) and \( r_{oi} \) represent the amount of oxygen consumed in the nitrification of ammonium and nitrite, respectively.

Solve these equations numerically with the forward Euler approach and make two graphs for a 20-day period: one with the concentrations of the nitrogen species as a function of time and another with the dissolved oxygen concentration as a function of time. Take \( k_{oa} = k_{ai} = 0.25 \text{ d}^{-1}, k_{in} = 0.75 \text{ d}^{-1}, k_a = 0.5 \text{ d}^{-1}, r_{oa} = 3.43 \text{ gO/gN}, r_{oi} = 3.43 \text{ gO/gN}, \) and \( DO_{sat} = 10 \text{ mg/L} \). For initial values, take \( N_o = N_a = 6 \text{ mg/L} \) and \( N_i = N_n = D = 0 \).