CE 473/573 Groundwater  
Fall 2009  
Comments on homework 4

21. No divide will occur when \( w_0/K < (h_i^2 - h^2_2)/L^2 \). The limiting recharge rate increases with \( K \) because the soil can conduct more water away, and it increases with increasing difference between the boundary heads because the flow rate through the aquifer is larger.

22. I compute a drain depth of about 0.9 m. Half the total flow per unit width (i.e., \( w_0L/2 \)) goes into each drain. For part c I get 5 mm/d

23. Conservation of mass and Darcy’s law applied to a numerical grid give

\[
h_i^2 = \frac{h_{i-1}^2 + h_{i+1}^2}{2} + \frac{w_0(\Delta x)^2}{K}
\]

Set the boundary values to \( h_0^2 \) and \( h_L^2 \) and use the above equation to iterate to find \( h^2 \). Then take the square root to compute the water table elevation.

24. For parts a-c, the water table is nearly linear, and the effective conductivity is essentially constant. For part d, split the aquifer into two parts: a left part \( (x < x_c, \text{ say}) \) and a right part \( (x > x_c) \). The right part is a homogeneous unconfined aquifer, and our solution from class can be modified so that the water table elevation is \( b_2 \) at \( x = x_c \) and \( h_L \) at \( x = L \):

\[
h = \left[ b_2^2 + (h_L^2 - b_2^2) \left( \frac{x - x_c}{L - x_c} \right) \right]^{1/2}
\]

The left part can be analyzed with the effective conductivity, as in part a. Then find \( x_c \) by requiring that the flows in the left and right parts are equal. I find \( x_c = 612.1 \text{ m} \) and \( Q' = 996.9 \text{ m}^2/\text{d} \).

25. To determine \( \lambda \), I guessed values, computed the heads at the given distances, and computed the sum of the squared differences between the measured and calculated heads. (Square the differences so that all are positive.) Then I used Solver in Excel to minimize the sum of squared differences. In this way, I found \( \lambda = 1.33 \times 10^{-3} \text{ m}^{-1} \).

The flow per unit width is

\[
Q' = -K \frac{dh}{dx}b = \frac{Kb\lambda}{\sinh \lambda L} [s_0 (\sinh \lambda x \sinh \lambda L - \cosh \lambda x \cosh \lambda L) + s_L \cosh \lambda x]
\]

I find \( Q' = 0.13 \text{ m}^2/\text{d} \). A groundwater divide will occur when \( Q' = 0 \), and since the minimum value of the flow will occur at \( x = 0 \), put \( x = 0 \) in the above equation and solve for \( H \), the head outside the confined aquifer, remembering that \( s_0 = H - h_0 \) and \( s_L = H - h_L \). I find \( H = 15.9 \text{ m} \).
27. Many groups used the analytical solution to determine the head at \( x = 800 \text{ m} \) and the usual numerical method to compute the heads in between. Although this approach works in this case, the proper approach is to set the head at \( x = 0 \) and compute the head at the next grid cell using Darcy’s law—i.e.,

\[
h_1 = h_0 - \frac{Q_0 \Delta x}{Kbw}.
\]

For the remaining points, equate the flows in and out of a cell to get

\[
h_{i+1} = 2h_i - h_{i-1}.
\]

Here the head in the downstream cell \((h_{i+1})\) is unknown.