Chapter 1

Review of Multivariable Calculus

1.1 Review of Partial Differentials and Chain Rule

1.1.1 Definition of Partial Derivatives

$$f_x(x_0, y_0) = \lim_{\Delta x \to 0} \left[\frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} \right]$$

$$f_y(x_o, y_0) = \lim_{\Delta y \to 0} \left[\frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} \right]$$

1.1.2 Properties of Partial Derivatives

$(f+g)_x = f_x + g_x$	$(f_x)_x = f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$
$(f-g)_x = f_x - g_x$	$(f_x)_y = f_{xy} = \dfrac{\partial^2 f}{\partial y \partial x}$
$(fg)x = f_x g + f g_x$	$(f_y)_x = f_{yx} = \dfrac{\partial^2 f}{\partial x \partial y}$
$(f/g)_x = \frac{f_x g - f g_x}{g^2}$	$f_{xy} = f_{yx}$

1.1.3 Chain Rule

1.

$$z = f(x, y)$$
$$x = g_1(t)$$
$$y = g_2(t)$$
$$z = f(g_1(t), g_2(t)) = z(x, y) = z(t)$$
$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

2.

$$z = f(x, y)$$
$$x = g_1(u, v)$$
$$y = g_2(u, v)$$

then $z = f(g_1(u, v), g_2(u, v)) = z(x, y) = z(u, v)$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

In 3-D

1.

$$w = f(x, y, z)$$
$$x = g_1(t)$$
$$y = g_2(t)$$
$$z = g_3(t)$$
$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

2.

$$x = g_1(u, v)$$
$$y = g_2(u, v)$$
$$z = g_3(u, v)$$
$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial y}{\partial v}$$