1. Show that Theorem 3 on page 301 in the text remains valid if the condition that $\Omega$ be bounded is replaced by the more general assumption that $\Omega$ lies between two parallel hyperplanes in $\mathbb{R}^N$. (Suggestion: You’ll want to show that the Poincaré inequality is still valid.)

2. Find the best constant in the Poincaré inequality

$$||u||_{L^2(\Omega)} \leq C||Du||_{L^2(\Omega)} \quad \text{for } u \in H^1_0(\Omega)$$

if $\Omega$ is the box

$$\Omega = \{ x \in \mathbb{R}^N : a_j < x_j < b_j \quad j = 1 \ldots N \}$$

or the ball $\Omega = B(z, R)$. 