In all of these problems $P(D)$ denotes a constant coefficient linear partial differential operator of order $m \geq 1$ in $\mathbb{R}^N$.

8. If $N = 1$ show that $P(D)$ is elliptic.

9. If $N > 1$, $m = 1$ and the coefficients are real, show that $P(D)$ is not elliptic.

10. If $P(D)$ is elliptic, show that $\lim_{|y| \to \infty} |P(y)| = \infty$ and that

$$N(P) = \{ y \in \mathbb{R}^N : P(y) = 0 \}$$

is compact.

11. Show that $P(D)Q(D)$ is hypoelliptic if and only if $P(D)$ and $Q(D)$ are separately hypoelliptic.

12. Do problem #19, page 249 in the text. Explain also why there is no contradiction between the second part of the problem and the fact there exist a solution of the initial value problem for the heat equation with $u(x, 0) = 1/(1 + x^2)$. 