6. Assuming that \( f'' > 0 \), find solutions of the conservation law \( u_t + (f(u))_x = 0 \) in the form \( u(x,t) = \theta(x/t) \) (these are sometimes called simple waves).

7. Let

\[
\begin{align*}
  u_1(x,t) &= \begin{cases} 
    1 & x < \frac{t}{2} \\
    0 & x > \frac{t}{2}
  \end{cases} \\
  u_2(x,t) &= \begin{cases} 
    1 & x < t \\
    0 & x > t
  \end{cases}
\end{align*}
\]

Show directly from the definition (page 136 in text, where the terminology 'integral solution' is used) that \( u_1 \) is weak solution of

\[
  u_t + uu_x = 0 \quad x \in \mathbb{R} \quad t > 0
\]

\[
u(x,0) = \begin{cases} 
  1 & x < 0 \\
  0 & x > 0
\end{cases}
\]

but \( u_2 \) is not.