35. A nonconstant solution $φ(t)$ for $x' = f(x)$ is called a homoclinic orbit if there exists $ξ_0 ∈ \mathbb{R}^n$ such that $\lim_{t \to +∞} φ(t) = \lim_{t \to -∞} φ(t) = ξ_0$. In this case we also say that $ξ_0$ is a homoclinic point. Such a point is necessarily a saddle point.

   a) Show that the origin is a homoclinic point for the system

   \[
   \begin{align*}
   x'_1 &= -x_1 x_2 \\
   x'_2 &= x_1 - x_2^2
   \end{align*}
   \]

   (Suggestion: Obtain $x_2$ as a function of $x_1$ by solving $\frac{dx_2}{dx_1} = (x_2^2 - x_1)/x_1 x_2$.)

   b) Show that for any $ε, M > 0$ there exists a solution $φ(t)$ such that $|φ(0)| < ε$, $\lim_{t \to +∞} φ(t) = 0$, but $|φ(t)| > M$ for some $t$.

   c) What does linearization tell you about the phase portrait near the origin?

36. Show that the zero solution of a linear, autonomous Hamiltonian system with Hamiltonian $H(q,p) = q^T S_1 q + p^T S_2 p$ can never be asymptotically stable. Here $S_1, S_2$ are constant matrices. Recall the definition of a Hamiltonian system from problem #13.

37. Discuss the phase portrait for the second order ODE

   \[ y'' + y - y^2 = 0 \]

   in as much detail as you can. (Find critical points, discuss their stability and note that it is possible to find an exact formula for the orbits in the phase plane. You can use something like pplane to get an idea of what is going on, but what I am looking for here is conclusions with mathematical justification.)