32. Find and classify by stability and type all critical points of the system

\[
x'_1 = 60x_1 - 3x_1^2 - 4x_1x_2 \quad x'_2 = 42x_2 - 3x_2^2 - 2x_1x_2
\]

33. Find the \textit{pplane} software for generating phase plane diagrams and figure out how to use it. (For example a Java version, pplane.jar, can be found at math.rice.edu/~dfield/dfpp.html) Use it to investigate the system

\[
x'_1 = -x_1 - x_2^2 \cos(x_1 + x_2) \quad x'_2 = x_2 + x_1^2 \cos(x_1 - x_2)
\]

There are lots of critical points, which you probably can’t find analytically, except for \( x = 0 \). So you should try to locate a number of these points numerically (\textit{pplane} will do this also), classify them, and give some idea about the structure of other orbits in the phase plane. Be sure to look at a reasonable size portion of the phase plane, say \([-5, 5] \times [-5, 5]\). Include a printout of the phase plane diagram produced by \textit{pplane}.

34. Consider the system

\[
x'_1 = \epsilon x_1 + x_2 - x_1(x_1^2 + x_2^2) \\
x'_2 = -x_1 + \epsilon x_2 - x_2(x_1^2 + x_2^2)
\]

where \( \epsilon \) is a parameter.

a) Introduce polar coordinates \((r, \theta)\) in the phase plane \((x_1 = r \cos \theta, x_2 = r \sin \theta)\) and find the system satisfied by \((r(t), \theta(t))\).

b) Show that the system has a periodic solution for \( \epsilon > 0 \). (The appearance of a periodic solution as the parameter \( \epsilon \) becomes positive is called a \textit{Hopf bifurcation}.)

c) Show that the origin is an asymptotically stable spiral point for \( \epsilon < 0 \) and an unstable spiral point for \( \epsilon > 0 \).