26. Define the translation operator \( Tu(x) = u(x - 1) \) on \( L^2(\mathbb{R}) \).
   a) Find \( T^* \).
   b) Show that \( T \) is unitary.
   c) Show that \( \sigma(T) = \sigma_c(T) = \{ \lambda \in \mathbb{C} : |\lambda| = 1 \} \).

27. Let \( Tu(x) = \int_0^x K(x, y)u(y) \, dy \) be a Volterra integral operator on \( L^2(0, 1) \) with a bounded kernel, \( |K(x, y)| \leq M \). Show that \( \sigma(T) = \{0\} \). (There are several ways to show that \( T \) has no nonzero eigenvalues. Here is one approach: Define the equivalent norm on \( L^2(0, 1) \)
   \[
   \|u\|_\theta^2 = \int_0^1 u^2(x)e^{-2\theta x} \, dx
   \]
   and show that the supremum of \( \frac{\|Tu\|_\theta}{\|u\|_\theta} \) can be made arbitrarily small by choosing \( \theta \) sufficiently large.)

28. Let \( T \) be the integral operator
   \[
   Tu(x) = \int_0^1 (x + y)u(y) \, dy
   \]
on \( L^2(0, 1) \). Find \( \sigma_p(T) \), \( \sigma_c(T) \) and \( \sigma_r(T) \) and the multiplicity of each eigenvalue.

29. Show that if \( S \in \mathcal{B}(H) \) and \( T \) is compact, then \( TS \) and \( ST \) are also compact. (In algebraic terms this means that the set of compact operators is an \textit{ideal} in \( \mathcal{B}(H) \).)

30. If \( T \in \mathcal{B}(H) \) and \( T^*T \) is compact, show that \( T \) must be compact. Use this to show that if \( T \) is compact then \( T^* \) must also be compact.