16. If $T$ is symmetric and $R(T) = H$ show that $T$ is self-adjoint. (Suggestion: the main thing to show is that $D(T^*) \subset D(T)$.)

17. Give a careful justification of the following fact stated in class: If $Tx = 0$ for $x \in D(T)$, where $D(T)$ is dense in $H$ but unequal to $H$ then $T$ is not closed.

18. If $T$ is self-adjoint, $S$ is symmetric and $T \subset S$, show that $T = S$. (Thus a self-adjoint operator has no proper symmetric extension).

19. (See problem 10.6 in text) Let $T, S$ be densely defined linear operators on $H$ and assume that $D(T + S) = D(T) \cap D(S)$ is also dense. Show that $T^* + S^* \subset (T + S)^*$. Give an example showing that $T^* + S^*$ and $(T + S)^*$ may be unequal.

20. (See problem 9.4 in text) Let $M$ be a closed subspace of a Hilbert space $H$, $M \neq \{0\}, H$. Show that if $\lambda \neq 0, 1$ then $\lambda \in \rho(P_M)$ ($P_M$ the usual orthogonal projection) and

$$|| (P_M - \lambda I)^{-1} || \leq \frac{1}{|\lambda|} + \frac{1}{|1 - \lambda|}$$