

Outline of sections covered on Exam 3

- 16.1 The double integral $\iint_R f(x, y) dA$ over rectangle R is defined as a limit of Riemann sums
- 16.2 To evaluate this double integral we convert it to an equivalent iterated integral $\int_a^b \int_c^d f(x, y) dydx$, and then use one variable integration techniques.

- 16.3 For integration over more complicated regions S we supply limits of integration which may themselves be functions, for example

$$\iint_S f(x, y) dA = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x, y) dydx$$

if the region S can be described by equations

$$\phi_1(x) \leq y \leq \phi_2(x), a < x < b$$

The order of integration may be reversed, providing we change the limits of integration in the appropriate way.

- 16.4 Transformation to polar coordinates

$$\iint_S f(x, y) dA = \int \int f(r \cos \theta, r \sin \theta) r drd\theta$$

with appropriate limits supplied, may make the integral simpler to evaluate.

- 16.5 If a planar region S has density function $\delta(x, y)$ then various physical properties such as total mass, center of mass, and moments of inertia may be computed by means of integrals over S involving δ .
- 16.6 The area of a surface $z = f(x, y)$ lying over a region S of the xy plane is

$$\iint_S \sqrt{f_x^2(x, y) + f_y^2(x, y) + 1} dA$$

- 16.7 Triple integrals in Cartesian coordinates may be computed by means of equivalent iterated integrals, such as

$$\iiint_S f(x, y, z) dV = \int_{a_1}^{a_2} \int_{\phi_1(x)}^{\phi_2(x)} \int_{\psi_1(x, y)}^{\psi_2(x, y)} f(x, y, z) dzdydx$$

- 16.8 Triple integrals may be transformed to cylindrical coordinates (r, θ, z) or spherical coordinates (ρ, ϕ, θ) using

$$dV = dx dy dz = r dr d\theta dz = \rho^2 \sin \phi d\rho d\phi d\theta$$

and appropriate limits.