Show your work! Do not write on this test page!

1. (30 points) Let $f(x,y) = \dfrac{2}{x^2+y}$.
   (a) What is the domain of f?
   (b) Sketch the level curve $f(x,y) = 1$.
   (c) Compute the partial derivatives $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ and $\dfrac{\partial^2 f}{\partial x \partial y}$.
   (d) Use the differential approximation to estimate the change in f as (x,y) changes from $(2,1)$ to $(2.1,.8)$.

Solution: a) All points (x,y) such that $y \neq -x^2$, b) the downward opening parabola $y = 2 - x^2$, c) $f_x = -4x(x^2+y)^{-2}$, $f_y = -2(x^2+y)^{-2}$, $f_{xy} = 8x(x^2+y)^{-3}$, d) -.016

2. (20 points) Let $f(x,y,z) = x^2z^3 - y + ze^x$.
   (a) In what direction is f increasing most rapidly at the point $P = (0,1,3)$?
   (b) Find the directional derivative of f at the point $(0,1,3)$ in the direction $< 1,-2,2 >$
   (c) Show that f has no critical points.

Solution: a) $< 3,-1,1 >$, b) 7/3, c) $f_y = -1$ so ∇f can never be the zero vector

3. (15 points) Find the equation of the tangent plane to the surface
$$z^2 - 2x^2 - 2y^2 = 12$$
at the point $(1,-1,4)$.

Solution: $-x + y + 2z = 6$

4. (15 points) A particle moves in the xy plane in such a way that at time $t = 1$ its x coordinate $x(t)$ satisfies $x(1) = 3, x'(1) = 1$ and the y coordinate $y(t)$ satisfies $y(1) = 4, y'(1) = 5$. At what rate is the distance of the particle from the origin changing when $t = 1$? (Suggestion: start by expressing the distance to the origin in terms of x and y.)

Solution: 23/5

5. (20 points) Using the Lagrange multiplier method, find the maximum value of x^2y on the circle $x^2 + y^2 = 3$, and the point or points at which the maximum value occurs.

Solution: The maximum value is 2, and occurs at $(\pm\sqrt{2}, 1)$