The total number of points for this exam is 22. You have 50 minutes to complete the exam. Do both problems. Show all your work so that I can see how you arrived at the answer. You may use Griffith “Introduction to Quantum Mechanics” and the lecture notes to look up the relevant formulas.

1. Consider a set of identical harmonic oscillators with the potential energy function $V(x) = \frac{1}{2}m\omega^2x^2$. Each oscillator is at time $t = 0$ in the normalized state

$$\psi_A \equiv \Psi_A(x, t = 0) = A(\psi_0 + 2\psi_1)$$

where the $\psi_n$ represent the stationary states.

(a) (2 points) Calculate $A$.

(b) (2 points) Give a linear combination $\psi_B$ of the stationary states $\psi_0$ and $\psi_1$ which is normalized and orthogonal to $\psi_A$.

(c) (2 points) If you measure the energy of a particle in state $\psi_A$ at time $t = 0$, what values can you measure? What is the probability of each?

(d) (2 points) Calculate the expectation value of the Hamiltonian $\langle H \rangle_A$ in state $\psi_A$.

(e) (2 points) Give $\Psi(x, t)_A$.

(f) (2 points) Calculate the expectation value of the momentum $\langle p \rangle_A$ in state $\Psi_A(x, t)$.

2. Consider a particle in the asymmetric infinite square well potential

$$V(x) = \begin{cases} \infty & \text{for } x \leq -a \text{ and } x > a \\ 0 & \text{for } -a < x \leq 0 \text{ (region I)} \\ V_0 & \text{for } 0 < x \leq a \text{ (region II)} \\ \end{cases}$$

(a) (4 points) Solve the Schrödinger equation in region I and II and give all boundary and matching conditions for the case $0 < E_A < V_0$. Do not attempt to solve for the coefficients or allowed energies.

(b) (4 points) Sketch qualitatively $|\psi_B|^2$ for a wave function $\psi_B$ that is a solution for the Schrödinger equation for the case $E_B > V_0 > 0$. Explain as many features as you can qualitatively.

(c) (2 points) If you measure the position of the particle in the state $\psi_B$, will you find it more likely at $x < 0$ or at $x > 0$? Explain.

Good luck!