Homework # 3, Population models,

Due: 5 pm, Friday, 20 November 2009, to my office (2121 Snedecor) or mailbox (in 1121 Snedecor)

Reminders: Choose **3 of the 4** problems. You are not expected to do all five. Also, we have not talked about assumptions or diagnostics for any of these models. You are not expected to do any here.

Problems 1 and 3 are 'data questions' that require executive summaries.

Problem 1.

Avicennia marina, the grey mangrove, is an abundant plant on mudflats in southern hemisphere estuaries (see photo linked to class homepage for pictures of *A. resinosa*, a related species in New Zealand). Moderate to large mangrove plants are very important ecologically because they anchor coastlines and are important nurseries for fish.

Mangrove forests can be disturbed in two different ways. Small-scale disturbances bring down one or a few mature trees. Large-scale disturbances (storms, disease, or sedimentation) create large areas of open habitat. P. J. Clarke in Australia collected demographic data on mangroves in plots with small-scale disturbances and in plots with large-scale disturbances.

Clarke used 7 stages to describe the demography and constructed transition matrices by following marked plants. The transition matrices describe the growth, survival and fecundity of those plants in habitats with the two types of disturbance. The data are in avicennia.txt in the data files section of the class web page. The survival and growth parts of these transition matrices are based on observations of many plants. The fecundity is much more uncertain. Using the limited information that is available about the uncertainty of each element in the transition matrix, I compute the uncertainty in λ as s.e. $\lambda \approx 0.07$.

Because most of the coastline in a particular National Park is subject to small-scale disturbances, folks are mostly interested in the demography in small-scale disturbance areas. Will the population grow? How uncertain is the estimated growth rate? How quickly will the population reach stable growth rate? Will the population contain a large fraction of moderate to large individuals? If a single large disturbance hits this coastline (so the dynamics for one year are driven by the large transition matrix), what is expected to happen to the population? What can you tell the Park managers?

Problem 2. Miscellaneous analytical issues.

2-a) Sensitivity to changes in multiple vital rates. Many stage-based transition matrices have the following form:

$$\begin{matrix} s_1(1-g_1) & f_2 & f_3 & f_4 \\ s_1 g_1 & s_2(1-g_2) & 0 & 0 \\ 0 & s_2 g_2 & s_3(1-g_3) & 0 \\ 0 & 0 & s_3 g_3 & s_4 \end{matrix}$$

where s_i is a stage-specific survival probability and g_i is a stage-specific conditional probability that a surviving plant will grow into the next stage. For many species, the sensitivity of λ to changes in s_i or g_i may be more important than the sensitivities we have computed (i.e., $\partial\lambda/\partial a_{ij}$). The sensitivities to s_i or g_i can be computed from the "usual" sensitivities. Derive the expressions to compute $\partial\lambda/\partial s_i$ and $\partial\lambda/\partial g_i$.
HINT: think about the chain rule for derivatives.

2-b) Confidence intervals for lambda. A long time ago, we talked about three ways to estimate a confidence interval for N: normal theory for N, normal theory for $\log N$, and a profile interval. The discussion about intervals for $\log N$ might be relevant to estimating an interval for λ.

Why might a standard (e.g. z- or t-based) interval for λ be problematic?
Work out a $(1-\alpha)$ confidence interval for λ based on the assumption that $\log \lambda$ has a normal distribution. This interval will need to be calculated from $\hat{\lambda}$ and $se\,\hat{\lambda}$.

Problem 3. Validating population estimates

Graduate student research is the source of a lot of demographic data. However, students tend to collect lots of data for (relatively) short periods of time. They often collect age- or stage-classified information. State Departments of Natural Resources are often required to monitor population sizes of rare or endanged plants and animals. They usually do not have the time to continue detailed demographic monitoring. Commonly, they only track the total population size over time. So, the following history of information about a specific population is not too uncommon:

Year	Type of data collected	Information available
1	Student starts study	Initial counts, i.e. N_1 and $\tilde{N}_1$
2	Student continues	N_2 and $\tilde{N}_2$
3	Student continues	N_3 and $\tilde{N}_3$
4	Student finishes	N_4 and $\tilde{N}_4$
5	DNR starts	N_5
6	DNR continues	N_6
7, 8, 9, 10	DNR continues	N_7, N_8, N_9, and N_{10}

where:
N_i is the total number of individuals in the population in year i, and
$\tilde{N}_i$ is the vector of counts of stage-classified individuals in year i

Imagine the student has collected data on a rare orchid, *Isotria*. They have estimated a pooled (over years 1-4) transition matrix, then used a deterministic matrix model to estimate $\hat{\lambda}$. The estimated $\hat{\lambda} = 1.051$ with se=0.023. The DNR has population counts for all 10 years:

Year	1	2	3	4	5	6	7	8	9	10
N	122	130	133	142	176	212	206	209	238	250

The DNR hires you, an eminent statistician, to tell them whether the population is continuing to grow at the rate seen by the graduate student. There are many techniques you could use. Decide what approach is most reasonable / appropriate. Think carefully about, and describe to the DNR as part of your report, the assumptions made when using your chosen approach.

Problem 4. Estimating the transition matrix from observed data. In class, I finished the discussion of population models by contrasting two approaches to analyze a time series of abundances. The 'process error only' approach used regression, conditioning on the numbers at the previous time. The 'observational error only' approach compared the observed counts to projections at each time. My illustration had 4 stages; to keep things simple, I will use only 2 stages (juvenile and adult) here. The appropriate transition matrix is:

$$\begin{bmatrix} 0 & f \\ s_j & s_a \end{bmatrix}$$

where:
s_j is survival of juveniles,
s_a is survival of adults, and
f is fecundity, number of surviving juveniles per adult

A population has been studied for 20 years; the counts of juveniles and adults are given in abdn2.txt.

Assume that all variation represents process error, so it is appropriate to condition on the number of individuals in the preceding generation. You may assume that process errors are normally distributed with constant variance. Please estimate the unknown coefficients in the transition matrix. Also estimate their standard errors.