Stat 402A, HW 2 answers.

1. **Sample size for survey of farmland value**
   This focus in this problem is estimating a mean. I tell you that $\sigma = 750$.

   (a) 3 pts. For a single sample mean, the s.e. $= \sigma/\sqrt{n}$, so we need to find n for which $100 = 750/\sqrt{n}$, i.e. $n = 56.25$. You need to sample $n = 57$ participants.

   Note: Please round sample sizes UP! You can't sample 0.25 individuals. Strictly speaking, 56 individuals does not satisfy the condition (s.e. < 100), so you need to round up.

   (b) 3 pts. The width of a 95% ci for a sample mean is $2\, t_{0.975,d.f.}\, \sigma/\sqrt{n}$. You need to find n so that this width $= 100$. One problem is that the t-quantile depends on the d.f., which depend on n. We solve this by iterating, just like in the power calculations. If we start with $t = 2$, we need to find n for which $100 = 2(2)(750/\sqrt{n})$, i.e. $n = 900$. That is sufficiently large to use the normal quantiles, i.e. $t_{0.975} = 1.96$, which gives $n = 864.36$, i.e. $n = 865$. You need to sample 865 people if the desired width of the 95% confidence interval is to be 100.

   Note: Anything close to 865 is acceptable, since different choice of "large" d.f. will give slightly different answers.

2. **Power for 2 sample comparisons (herbal extract and control).**

   (a) 2 pts. I tell you that $\sigma = 4.1$. The s.e. of the difference between two equal-sized samples is $\sigma\,\sqrt{2/n}$. Hence, for $n = 5$, the s.e. is $4.1\sqrt{2/5} = 2.59$.

   Note: σ is presumed to be the same for all groups, so you already have the pooled s.d.

   (b) 2 pts. The d.f. of a pooled estimate of s.d. are $n_1 - 1 + n_2 - 1 = 2(n-1) = 8$. The s.e. has 8 d.f. The appropriate quantile is 2.306.

   (c) 4 pts. Starting from the general power equation,

   $$\delta = (t_{1-\alpha/2} + t_\beta) s.e.,$$

   you know $\delta = 6.5$, α and d.f. (and so $t_{0.975,8} = 2.306$), and the s.e. $= 2.59$. So

   $$6.5 = (2.306 + t_\beta)2.59$$

   Doing a bit of rearranging and calculating gives you $t_\beta = 6.5/2.59 - 2.306 = 0.204$. This is the value you would use in a typical t-table (e.g. the one in the Statistical Sleuth). That table gives you P[value < 0.204], i.e. the left tail value. Using a t-distribution calculator with df $= 8$ gives you $\beta = 0.58$, using the "left tail". The proposed study has a power of 58%.

   Notes: This was probably the hardest question on this HW. We won't do this sort of calculation very often.

   I agree figuring out which tail to use is confusing. If you draw a picture (see below), the decision is a lot clearer. When the proposed difference, δ, leads to a t quantile between

0 and $t_{1-\alpha/2}$, the power is less than 50should match that. When the proposed difference, δ, leads to a t quantile beyond $t_{1-\alpha/2}$ (almost all the examples in class), the power is more than 50

If you prefer pictures, here's how to think about the problem. My pictures have three parts: the distribution under the null hypothesis, the point above which you reject the null hypothesis, and the distribution under the alternative hypothesis (i.e. based on the true difference, δ). The distribution under the null is a t distribution centered at 0. The point at which you reject H0 is $|t| = 2.306$ on the t scale or a difference of $|t * s.e.| = 5.97$ on the d scale. I give you $\delta = 6.5$, so the alternative distribution is a t-distribution centered at a difference of 6.5. Approximately 2/3 of this distribution is above 5.97 and approximately 1/3 is below, so the power is approximately 66%.

(d) 6 pts. I give you $\alpha = 0.05$ and $\beta = 0.90$. Substituting the s.e. for a 2-sample test (given above) into the general power equation

$$\delta = (t_{1-\alpha/2} + t_\beta)s.e. \tag{1}$$

and solving for n gives the formula derived in class:

$$n \geq 2\left(t_{1-\alpha/2} + t_\beta\right)^2 \frac{\sigma^2}{\delta^2}$$

The t quantiles aren't known, but reasonable starting values are $t_{0.975} = 2$ and $t_{0.90} = 1.3$. Lots of other starting values are equally reasonable. Substituting and calculating gives

$$n \geq 2\,(2 + 1.3)^2 \frac{4.1^2}{6.5^2} = 8.67$$

i.e. $n = 9$ per group. A two sample study with 9 per group = 18 total samples has 16 d.f., for which the t quantiles are 2.120 and 1.337, which gives a new sample size of

$$n \geq 2\,(2.120 + 1.337)^2 \frac{4.1^2}{6.5^2} = 9.51$$

i.e. $n = 10$. A two sample study with 10 per group=20 total samples has 18 d.f., for which the t quantiles are 2.101 and 1.330, which gives a new sample size of

$$n \geq 2\,(2.101 + 1.330)^2 \frac{4.1^2}{6.5^2} = 9.367$$

i.e. $n = 10$ per group. You don't need to iterate any more times when you get the same answer twice in a row.

The required number of replicates is $n = 10$ per group to obtain an 90% power to detect a difference of 6.5, using a two-sample t-test with $\alpha = 0.05$ when the population s.d. = 4.1.