Stat 402A, Spring 2009 - Degrees of freedom calculations for split plot experiments

Alfalfa cutting experiment:
Main plots: Variety of alfalfa (3 levels), RCB, 6 blocks
Split plots: Date of cutting (4 levels), CRD, 1 rep per main plot

Source	d.f.	MS	Var. comp	E MS
block*V	10	0.136	$\hat{\sigma}^2_m = 0.027$	$\sigma^2_s + q\sigma^2_m$
error	45	0.028	$\hat{\sigma}^2_s = 0.028$	σ^2_s

Where do error d.f. come from and why are some not integers?

Some s.e.'s are easy, because a MS estimates the needed variability. d.f. come from the MS

Quantity	Variance	Est. by	d.f.
Diff btwn split means	$2\sigma^2_s/mp$	2 MSE$/mp$	45
Diff btwn main means	$2(\sigma^2_s + q\sigma^2_m)/mq$	2 MS B*V$/mq$	10

Other s.e.'s are not easy. require adding or subtracting MS

Quantity	Variance	Est. by	d.f.
Split mean	$(\sigma^2_s + \sigma^2_m)/mp$	$((q-1)\text{MSE} + \text{MS B*V})/mpq$	??

Why $((q-1)\text{MSE} + \text{MS B*V})/mpq$? Because the expected value of that sum of MS is what we need
$((q-1)\text{ EMS E} + \text{ EMS B*V})/mpq = ((q-1)\sigma^2_s + (\sigma^2_s + q\sigma^2_m)/mpq = (q\sigma^2_s + q\sigma^2_m)/mpq = (\sigma^2_s + \sigma^2_m)/mp$

So what is the d.f. for $(q-1)\text{MSE} + \text{MS B*V})$?? 3 options:

Containment (SAS default): determine which effects are main and which are split; use main plot error df (10) for main plot effects and split plot error df (45) for split plot effects. Often reasonable but is too large for some pairwise differences. Usually equivalent to another conservative approach: use smaller of df for MSE and df for MS B*V.

Satterthwaite approximation (1941, Psychometrika): d.f. for $c_1\text{MS}_1 + c_2\text{MS}_2$: df $= \frac{(c_1\text{MS}_1 + c_2\text{MS}_2)^2}{\frac{(c_1\text{MS}_1)^2}{\text{df}_1} + \frac{(c_2\text{MS}_2)^2}{\text{df}_2}}$

Check: MS main = 0.1362 with 10 d.f. MS split = 0.0279 with 45 d.f.
d.f. for 3 MS split + MS main = $= \frac{(3*0.0279+0.1362)^2}{(3*0.0279)^2/45+(1*0.1362)^2/10} = \frac{0.2199^2}{0.000156+0.001855} = 24.06$
SAS gives 24.1

Kenward-Roger approximation (1997, Biometrics): Formulae are complicated matrix expressions
Results (tests, se's, ci's) same as Satterthwaite for balanced split plots
Very close to Satterthwaite for unbalanced split plots
Applicable to more complicated designs (e.g. repeated measures with autocorrelation)
adjusts both the variance and the d.f.
p-values correctly calculated, except for small samples and very complex designs.
e.g. with complex repeated measures design, stated p-value might be 4%, but real value might be 12%.
But, everything else is even worse.