Stat 402A: models for correlations between repeated measures

Some covariance matrices for three repeated measures per e.u.

Independence
1 parameter: σ^2

$$\begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

Split-plot, Compound Symmetry
2 parameters: σ^2_m, σ^2_s

$$\begin{bmatrix} \sigma^2_m + \sigma^2_s & \sigma^2_m & \sigma^2_m \\ \sigma^2_m & \sigma^2_m + \sigma^2_s & \sigma^2_m \\ \sigma^2_m & \sigma^2_m & \sigma^2_m + \sigma^2_s \end{bmatrix}$$

AR(1)
2 parameters: σ^2, ρ

$$\begin{bmatrix} \sigma^2 & \rho\,\sigma^2 & \rho^2\,\sigma^2 \\ \rho\,\sigma^2 & \sigma^2 & \rho\,\sigma^2 \\ \rho^2\,\sigma^2 & \rho\,\sigma^2 & \sigma^2 \end{bmatrix}$$

AR(1) + RE
3 parameters: σ^2_m, σ^2, ρ

$$\begin{bmatrix} \sigma^2_m + \sigma^2 & \sigma^2_m + \rho\,\sigma^2 & \sigma^2_m + \rho^2\,\sigma^2 \\ \sigma^2_m + \rho\,\sigma^2 & \sigma^2_m + \sigma^2 & \sigma^2_m + \rho\,\sigma^2 \\ \sigma^2_m + \rho^2\,\sigma^2 & \sigma^2_m + \rho\,\sigma^2 & \sigma^2_m + \sigma^2 \end{bmatrix}$$

ARH(1)
4 parameters: σ^2_1, σ^2_2, σ^2_3, ρ

$$\begin{bmatrix} \sigma^2_1 & \rho\,\sigma_1\sigma_2 & \rho^2\,\sigma_1\sigma_3 \\ \rho\,\sigma_1\sigma_2 & \sigma^2_2 & \rho\,\sigma_2\sigma_3 \\ \rho^2\,\sigma_1\sigma_3 & \rho\,\sigma_2\sigma_3 & \sigma^2_3 \end{bmatrix}$$

ARH(1) + RE
5 parameters: σ^2_m, σ^2_1, σ^2_2, σ^2_3, ρ

$$\begin{bmatrix} \sigma^2_1 + \sigma^2_m & \rho\,\sigma_1\sigma_2 + \sigma^2_m & \rho^2\,\sigma_1\sigma_3 + \sigma^2_m \\ \rho\,\sigma_1\sigma_2 + \sigma^2_m & \sigma^2_2 + \sigma^2_m & \rho\,\sigma_2\sigma_3 + \sigma^2_m \\ \rho^2\,\sigma_1\sigma_3 + \sigma^2_m & \rho\,\sigma_2\sigma_3 + \sigma^2_m & \sigma^2_3 + \sigma^2_m \end{bmatrix}$$

ANTE(1)
5 parameters: σ^2_1, σ^2_2, σ^2_3, ρ_1, ρ_2

$$\begin{bmatrix} \sigma^2_1 & \rho_1\,\sigma_1\sigma_2 & \rho_1\rho_2\,\sigma_1\sigma_3 \\ \rho_1\,\sigma_1\sigma_2 & \sigma^2_2 & \rho_2\,\sigma_2\sigma_3 \\ \rho_1\rho_2\,\sigma_1\sigma_3 & \rho_2\,\sigma_2\sigma_3 & \sigma^2_3 \end{bmatrix}$$

UN
6 parameters: σ^2_1, σ^2_2, σ^2_3, σ_{12}, σ_{23}, σ_{13}

$$\begin{bmatrix} \sigma^2_1 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma^2_2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma^2_3 \end{bmatrix}$$

Corresponding correlation models

Independence

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Split-plot

$$\begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}$$

AR(1)

$$\begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}$$

AR(1) + RE

$$\begin{bmatrix} 1 & f + (1-f)\,\rho & f + (1-f)\,\rho^2 \\ f + (1-f)\,\rho & 1 & f + (1-f)\,\rho \\ f + (1-f)\,\rho^2 & f + (1-f)\,\rho & 1 \end{bmatrix}$$

ARH(1)

$$\begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}$$

ARH(1) + RE

not nice

ANTE(1)

$$\begin{bmatrix} 1 & \rho_1 & \rho_1\rho_2 \\ \rho_1 & 1 & \rho_2 \\ \rho_1\rho_2 & \rho_2 & 1 \end{bmatrix}$$

UN

$$\begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}$$

Summary of characteristics of correlation models

Model	Variances	Correlations
Independence	constant	none
Split-plot	constant	same for all pairs
AR(1)	constant	decline with separation
		eventually zero
		same for 1 to 2 and 2 to 3
AR(1) + RE	constant	decline to non-zero
ARH(1)	different	decline with separation
ARH(1) + RE	different	decline to non-zero
ANTE(1)	different	decline to zero
		different for 1 to 2 and 2 to 3
UN	different	every pair different

Choosing an appropriate correlation structure

Model	# param	AIC	AICc	BIC
Independence	1	342.0	342.1	343.5
Split-plot	2	337.0	337.4	338.5
AR(1)	2	**333.1**	**333.5**	**334.6**
AR(1)+RE	3	$\hat{\sigma}^2_m = 0$, so same as AR(1)		
ARH(1)	4	335.8	337.2	338.2
ARH(1)+RE	5	$\hat{\sigma}^2_m = 0$, so same as ARH(1)		
ANTE(1)	5	335.7	338.9	340.3
UN	6	335.7	338.9	340.3

Effect of correlation structure on results for asparagus study

Model	Trt A - Trt B		Year 2 - Year 3	
	s.e.	d.f.	s.e.	d.f.
Independence	12.09	33	10.46	33
Split-Plot	17.84	9	7.82	24
AR(1)	18.34	9.3	6.98	23.1
ARH(1)	18.01	8.53	6.21	20.6
ANTE(1)	18.48	8.74	7.64	12
UN	18.62	8.29	7.49	12

Assuming independence over time produces clearly different results. **clearly wrong!**

Choice of model for correlated data has much less impact on results. Models with lots of parameters, e.g. UN, ANTE(1), have smaller d.f. and larger s.e. This is a general pattern. Results in loss of power. Multivariate approach corresponds to UN; also lower power.

Two rep. meas. models with similar AIC and BIC values will usually (but not always) produce similar (but not same) conclusions about treatment effects (F statistics, p-values, s.e. d.f.).

My advice: use AIC and biological knowledge to choose correlation model. Try to get close to a reasonable model. Don't worry too much about getting the model exactly right.