Stat 402: Miscellaneous topics in sample size

Why equal sample sizes?

Consider $n_1 + n_2 = 20$, $\sigma^2 = 1$:

n_1	n_2	s.e. of difference
2	18	0.67
6	14	0.44
8	12	0.41
9	11	0.404
10	10	0.402
11	9	0.404

When do I specifically consider unequal n's?

- many treatments, each compared to a single control. Increase # reps for control.
  e.g. 9 trt and control, $\sigma = 1$.

# for control	# for trt	total # e.u.	s.e. for C - T	s.e. for T - T
3	3	30	0.82	0.82
6	3	33	0.71	0.82
4	4	40	0.71	0.71

- When some treatments are more expensive than others, limit is on total cost.

  e.g. landscape effects of K fertilization on pine forest.
  ca \$200,000 per treated e.u. ca \$10,000 per control e.u.
  Can only afford 2 treated e.u.

# trt	# control	cost	s.e.
2	2	\$420,000	1.00 σ
2	4	\$440,000	0.87 σ
2	8	\$480,000	0.80 σ
2	20	\$600,000	0.74 σ
3	3	\$630,000	0.81 σ

  Assumes equal variances in treatment and control groups!

Effect of plot size on sd between plots.

Fairfield-Smith 1938, wheat yield

Plot size	area	s.d. of yield/acre
1 x 1	1	46.1
2 x 2	4	29.4
1 x 4	4	24.7
3 x 3	9	23.9

Fairfield-Smith's "law":
Given plots of area A_1 with s.d. s_1, can approximate s.d. of plots of area A_2:

$$s_2 = s_1 (A_1/A_2)^{(b/2)}$$

empirically, b between 0.2 and 0.8, often 0.5
F-S defined b in terms of variance, formula here is for s.d.Ṫhat's why exponent is $b/2$.

Examples:

Preliminary data A_1	s_1	New plots A_2	s_2	s.d. =
1	46.1	4	$46.1\,(1/4)^{0.25}$	37.6
5	35.3	9	$35.3\,(5/9)^{0.25}$	30.5
20	0.79	10	$0.79\,(20/10)^{0.25}$	0.94