Stat 402A: 3 way factorial treatment designs

Concepts from 2 way extend to more 3, 4, or more factors.

Complete factorial: all combinations of levels used as treatments.
Can be many treatments. e.g. expt on pig feed composition: 4 levels of lysine, 3 levels of methionine and 2 levels of protein: 24 treatments, 2 reps per treatment, 48 observations

Effects defined just as before:
marginal means are averages over "left out" factors and replicates
marginal means for lysine: averages over all levels of methionine, all levels of protein, and all replicates
marginal means for lysine*protien: averages over all levels of methionine and all replicates
cell means for M*L*P: averages over all replicates

Compute marginal means by averaging cell means (works for balanced or unbalanced data)

skeleton ANOVA table (I x J x K factorial, n reps in CRD)

Source	d.f.	d.f. in general
Treatments	23	IJK - 1
Lysine	3	I-1
Methionine	2	J-1
Protein	1	K - 1
L*M	6	(I-1)(J-1)
L*P	3	(I-1)(K-1)
M*P	2	(J-1)(J-1)
something else	6	(I-1)(J-1)(K-1)
Error	24	IKJ(n-1)

What is the something else? Answer: the 3 way interaction of L*M*P.

Interpretation of 3 way interaction:
1) A problem. (see below for possible solutions)
2) Answer to the Q: Are the two way interaction effects the same for each level of the third factor?

Remember 2 way interaction: Are the simple effects of A the same at every level of B?
3 way interaction generalizes this: Are the 2way A*B int. effects the same for each level of C?

Example: row 3 on graphs has no 3 way interaction. Why? The cell means:

level of C	level of B	Cell mean for: A=1	A=2
1	1	6	6
1	2	1	3
2	1	2	4
2	2	1	5

When C = 1, the interaction effect for A*B is (simple effect of A when B=1) - (simple effect of A when B=2) = (6 - 6) - (3 - 1) = -2
When C = 2, the interaction effect for A*B is (4 - 2) - (5 - 1) = -2
These are the same, so there is no 3 way interaction.

4 way interaction (if four factors): Are the 3 way A*B*C interaction effects the same for each level of D?

Why no A*B interaction in the fourth set of plots:
2 way interaction is a comparison of averages over replicates and levels of C. The cell means and the averages of C=1 and C=2:

level of C	level of B	Cell mean for: A=1	A=2
1	1	3	7
1	2	3	3
2	1	4	4
2	2	0	4
ave.	1	3.5	5.5
ave.	2	1.5	3.5

Effect of A when B=1 is 5.5 - 3.5 = 2, when B=2 is 3.5 - 1.5 = 2. No A*B interaction

What to do if 3 way interaction is significant?
1) is additive model appropriate? multiplicative effects?
2) split data, describe L and M effects at each level of P,
or P and M effects at each level of L
or P effects at each level of L and M
Use 2 way interactions to decide how to split. Goal is simple summary of treatment effects
3) ignore. If magnitude small, may decide to use marginal mean as a single approximation.

Concepts extend to many factors. Commonly see 4 or 5 factor designs. Have seen 15.

Watch out for # treatments.
Industrial screening studies often 2 levels of each treatment. 8 factor study has 256 treatments: 8 main effects and lots of information about lots of interactions.
28 2 way interactions, 56 3 way interactions, ..., 1 8 way interaction

Often (not always) magnitude of main effect > that of 2 way interactions > 3 way > ... > 8 way interaction

Most interested in main effects and (perhaps) 2 way interactions. May be reasonable to assume high order interactions (e.g. 4 way, 5 way, ... 8 way) are zero. If so, can reduce the number of treatments and still get good estimates of main effects and some interactions.

Fractional factorials (no details, see me if want to consider)
Study 3 factors, 2 levels each (+ or -), 8 treatments. A 1/2 fraction estimates main effects, no interactions from 4 treatments.

Trt	A	B	C
1	-	-	+
2	-	+	-
3	+	-	-
4	+	+	+
omit	-	-	-
omit	-	+	+
omit	+	-	+
omit	+	+	-

Notice each trt occurs twice and is absent twice. Estimates of the main effects are more precise than if you used a 'one at a time' design (also four treatments).

Trt	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	-	-	+