

The sample variance and sample standard deviation: s^2 & s

The sample variance s^2 and the sample standard deviation s provide an important measures of variation or spread in a set of data. The sample variance, loosely speaking, is the “average” squared distance from a data value (say y_i) to the sample average \bar{y} of all data values; that is, the sample variance

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n - 1}$$

is an average of the squared deviation across all values y_1, y_2, \dots, y_n in the data set. (Recall \bar{y} again may be used as a measure of the central or typical value of the data, so with $(y_i - \bar{y})^2$ we access how spread the value y_i is away from the center of the data \bar{y}). The more distant data values are from the sample mean \bar{y} of the data, the larger the sample variance s^2 and the large the sample standard deviation $s = \sqrt{s^2}$.

You should have learned how to compute the standard deviation of a set of numbers in your introductory statistics class. In case you have forgotten or never learned, the example below shows how to compute the standard deviation of a small data set. Usually we use a computer to find standard deviations, but you should be able to compute the standard deviation using your calculator.

Example of Steps in Computing s^2 and s

Given data: 1,3,8,12 (Note: number of values $n = 4$)

1. Find sample mean $\bar{y} = \sum_{i=1}^n y_i/n$: $\bar{y}=(1+3+8+12)/4=6$
2. Find deviations from sample mean: $(y_i - \bar{y})$ for each value y_i
1-6=-5
3-6=-3
8-6=2
12-6=6
3. Square the deviations from sample mean: $(y_i - \bar{y})^2$ for each value y_i
 $(-5)^2=25$
 $(-3)^2=9$
 $2^2=4$
 $6^2=36$
4. Sum the squared deviations: $\sum_{i=1}^n (y_i - \bar{y})^2$
 $25+9+4+36=74$
5. Divide the result by one less than the number of observations ($n - 1$) to get $s^2 = \sum_{i=1}^n (y_i - \bar{y})^2 / (n - 1)$:
 $s^2=74/(4-1)=24.66666666$ (sample variance)
6. Take the square root of s^2 to get the standard deviation s :
 $s = \sqrt{s^2} = \sqrt{(24.66666666)}=4.967$ (sample standard deviation)