from **residuals.sas**

$\mu(Y_i \mid X_i) = \beta_0 + \beta_1 X_i = 5 + 2X_i$

Obs	X	muY	e	Y
1	2	9	0.86	9.86
2	2	9	0.29	9.29
3	2	9	0.00	9.00
4	3	11	-1.96	9.04
5	3	11	-0.19	10.81
6	3	11	-0.96	10.04
7	4	13	0.83	13.83
8	4	13	0.36	13.36
9	4	13	0.66	13.66
10	5	15	-1.11	13.89
11	5	15	-1.12	13.88
12	5	15	0.14	15.14
13	6	17	-1.37	15.63
14	6	17	-0.35	16.65
15	6	17	1.18	18.18
16	7	19	-0.44	18.56
17	7	19	-0.10	18.90
18	7	19	1.18	20.18

Suppose $\beta_0 = 5, \beta_1 = 2$

Then, we can find
$\mu(Y_i \mid X_i) = \beta_0 + \beta_1 X_i$
for each X_i
& each $Y_i = \mu(Y \mid X_i) + e_i$

In practice, we DO NOT know
$\beta_0, \beta_1, \mu(Y_i \mid X_i), e_i$

These need to be estimated only from 18 pairs (X_i, Y_i)

In practice, we would only be given or know the 18 pairs (X_i, Y_i)

```
proc reg;
  model Y=X;
  output out=two residual=ehat predicted=yhat;
run;

proc univariate plot;
  var ehat;
run;

proc plot;
  plot ehat*yhat;
run;
```

The REG Procedure
Model: MODEL1
Dependent Variable: Y

Number of Observations Read	18
Number of Observations Used	18

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	212.44709	212.44709	240.75	<.0001
Error	16	14.11891	0.88243		
Corrected Total	17	226.56600			

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	4.83105	0.62401	7.74	<.0001
X	1	2.01162	0.12965	15.52	<.0001

X_i → X $\mu(Y_i \mid X_i)$ → muY e_i → e Y_i → Y $\hat{Y}_i = 4.83 + 2.01X_i$ → roundedyhat $\hat{e}_i = Y_i - \hat{Y}_i$ → roundedresid

Obs	X	muY	e	Y	yhat	ehat	roundedyhat	roundedresid
1	2	9	0.86	9.86	8.8543	1.00571	8.85	1.01
2	2	9	0.29	9.29	8.8543	0.43571	8.85	0.44
3	2	9	0.00	9.00	8.8543	0.14571	8.85	0.15
4	3	11	-1.96	9.04	10.8659	-1.82590	10.86	-1.82
5	3	11	-0.19	10.81	10.8659	-0.05590	10.86	-0.05
6	3	11	-0.96	10.04	10.8659	-0.82590	10.86	-0.82
7	4	13	0.83	13.83	12.8775	0.95248	12.87	0.96
8	4	13	0.36	13.36	12.8775	0.48248	12.87	0.49
9	4	13	0.66	13.66	12.8775	0.78248	12.87	0.79
10	5	15	-1.11	13.89	14.8891	-0.99914	14.88	-0.99
11	5	15	-1.12	13.88	14.8891	-1.00914	14.88	-1.00
12	5	15	0.14	15.14	14.8891	0.25086	14.88	0.26
13	6	17	-1.37	15.63	16.9008	-1.27076	16.89	-1.26
14	6	17	-0.35	16.65	16.9008	-0.25076	16.89	-0.24
15	6	17	1.18	18.18	16.9008	1.27924	16.89	1.29
16	7	19	-0.44	18.56	18.9124	-0.35238	18.90	-0.34
17	7	19	-0.10	18.90	18.9124	-0.01238	18.90	0.00
18	7	19	1.18	20.18	18.9124	1.26762	18.90	1.28

The UNIVARIATE Procedure
Variable: ehat (Residual)

Extreme Observations

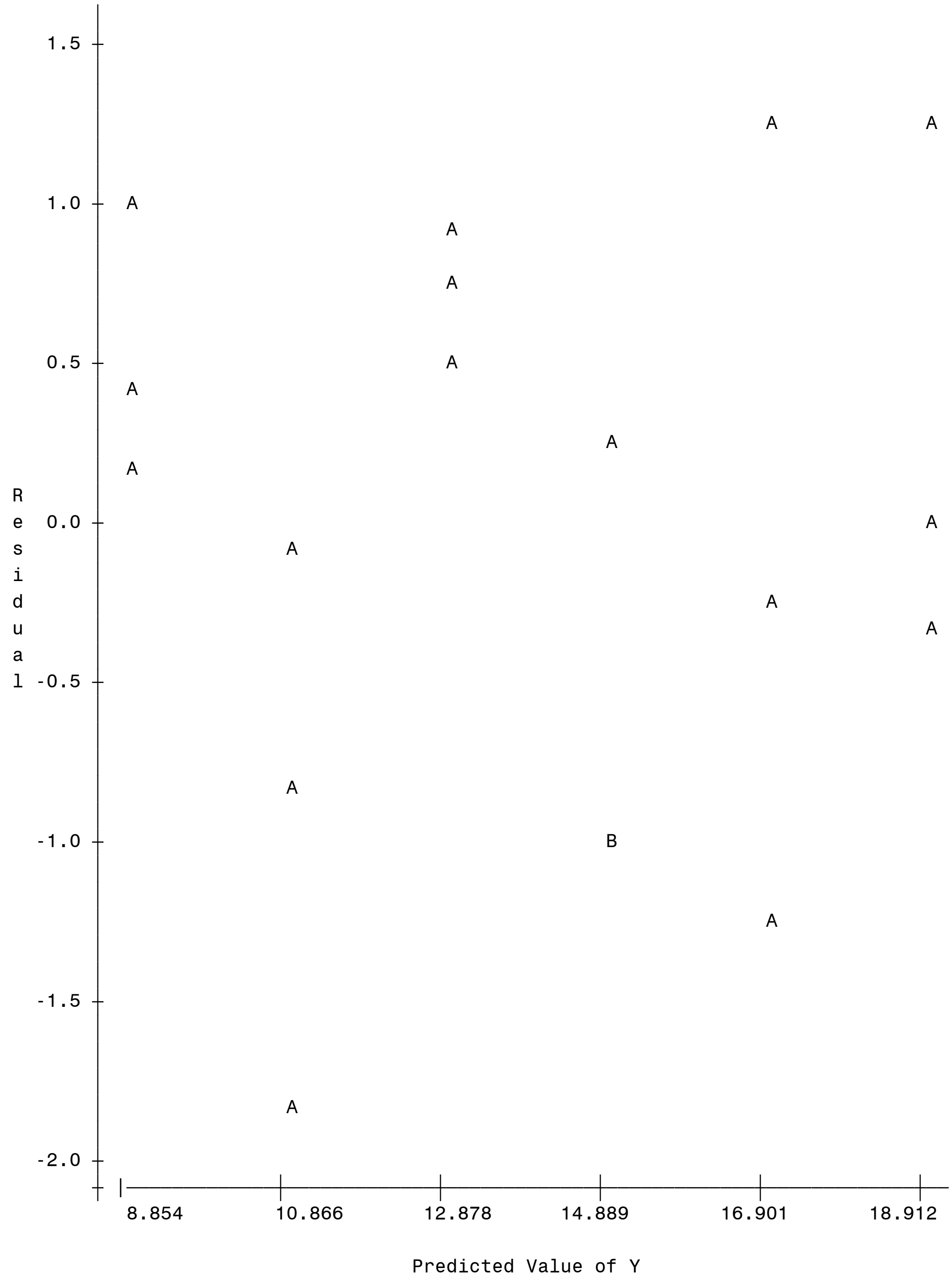
------Lowest------		------Highest-----	
Value	Obs	Value	Obs
-1.825905	4	0.782476	9
-1.270762	13	0.952476	7
-1.009143	11	1.005714	1
-0.999143	10	1.267619	18
-0.825905	6	1.279238	15

```
Stem Leaf                     #             Boxplot
   1 0033                     4                |
   0 58                       2             +-----+
   0 134                      3             *--+--*
  -0 4310                     4             |     |
  -0 8                        1             +-----+
  -1 300                      3                |
  -1 8                        1                |
     ----+----+----+----+
```

```
                     Normal Probability Plot
 1.25+                                        *+*+++ *
     |                                   +**++
     |                             +**+**
-0.25+                        **+**
     |                   ++**+
     |             +++*+*
-1.75+        +++*+
      +----+----+----+----+----+----+----+----+----+----+
          -2        -1         0        +1        +2
```

The SAS System

Plot of ehat*yhat. Legend: A = 1 obs, B = 2 obs, etc.



Note: if we take a different sample of size n=18, keeping the same X's, we could get different Y's

Again $\beta_0 = 5, \beta_1 = 2$ has NOT changed, although the sample is different

Then, we can find $\mu(Y \mid X_i) = \beta_0 + \beta_1 X_i$ for each X_i which does NOT change

In practice, we DO NOT know $\beta_0, \beta_1, \mu(Y \mid X_i), e_i$

These need to be estimated only from 18 pairs (X_i, Y_i)

Obs	X	muY	e	Y
1	2	9	1.76	10.76
2	2	9	-0.25	8.75
3	2	9	0.64	9.64
4	3	11	-0.21	10.79
5	3	11	0.68	11.68
6	3	11	0.03	11.03
7	4	13	1.54	14.54
8	4	13	-1.68	11.32
9	4	13	1.10	14.10
10	5	15	0.20	15.20
11	5	15	0.66	15.66
12	5	15	0.25	15.25
13	6	17	0.72	17.72
14	6	17	-0.38	16.62
15	6	17	-0.26	16.74
16	7	19	1.92	20.92
17	7	19	-0.60	18.40
18	7	19	-1.60	17.4

In practice again, we would only be given or know the 18 pairs (X_i, Y_i)

Note that the Y_i values & errors e_i are different since samples may vary

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	5.81940	0.67220	8.66	<.0001
X	1	1.87371	0.13966	13.42	<.0001

Different sample gives different estimates: $\hat{\beta}_0 = 5.82, \hat{\beta}_1 = 1.87$

X_i $\quad \mu(Y_i \mid X_i)$ $\quad e_i$ $\quad Y_i$ $\quad \hat{Y}_i = 5.82 + 1.87X_i$ $\quad \hat{e}_i = Y_i - \hat{Y}_i$

Obs	X	muY	e	Y	yhat	ehat	roundedyhat	roundedresid
1	2	9	1.76	10.76	9.5668	1.19317	9.56	1.20
2	2	9	-0.25	8.75	9.5668	-0.81683	9.56	-0.81
3	2	9	0.64	9.64	9.5668	0.07317	9.56	0.08
4	3	11	-0.21	10.79	11.4405	-0.65054	11.43	-0.64
5	3	11	0.68	11.68	11.4405	0.23946	11.43	0.25
6	3	11	0.03	11.03	11.4405	-0.41054	11.43	-0.40
7	4	13	1.54	14.54	13.3143	1.22575	13.30	1.24
8	4	13	-1.68	11.32	13.3143	-1.99425	13.30	-1.98
9	4	13	1.10	14.10	13.3143	0.78575	13.30	0.80
10	5	15	0.20	15.20	15.1880	0.01203	15.17	0.03
11	5	15	0.66	15.66	15.1880	0.47203	15.17	0.49
12	5	15	0.25	15.25	15.1880	0.06203	15.17	0.08
13	6	17	0.72	17.72	17.0617	0.65832	17.04	0.68
14	6	17	-0.38	16.62	17.0617	-0.44168	17.04	-0.42
15	6	17	-0.26	16.74	17.0617	-0.32168	17.04	-0.30
16	7	19	1.92	20.92	18.9354	1.98460	18.91	2.01
17	7	19	-0.60	18.40	18.9354	-0.53540	18.91	-0.51
18	7	19	-1.60	17.40	18.9354	-1.53540	18.91	-1.51