A Contrast for a Linear Trend

Sometimes populations or treatment groups are defined by levels of a quantitative or numeric variable, x (called an explanatory variable)

Example 1: $I = 3$ treatments defined by drug amount administered

Treatment	Drug Amount	Explanatory Variable (x)
Trt 1 (control)	0 mg	$x_1 = 0$
Trt 2	2 mg	$x_2 = 2$
Trt 3	8 mg	$x_3 = 8$

Example 2: amount of fertilizer $\Rightarrow$ 10, 20, 30, or 40 units of fertilizer per plot (conduct an experiment with 7 plots per treatment)

Treatment	Fertilizer Amount	Explanatory Variable (x)
Trt 1	10	$x_1 = 10$
Trt 2	20	$x_2 = 20$
Trt 3	30	$x_3 = 30$
Trt 4	40	$x_4 = 40$

• When I different levels of a quantitative variable $(x_1, \ldots, x_I)$ define I groups, we conduct an F-test as usual to test $H_0 : \mu_1 = \cdots = \mu_I$.
If the F-test turns out to be significant, it may be interesting to ask:

does the mean response μ_i vary or change with x_i?

• Specifically, we often investigate if there is a linear trend of the form:

$$\mu_i = \beta_0 + \beta_1 x_i$$

This implies that a straight line relationship exists between the level of x_i and the mean μ_i of the corresponding group or that the pairs $(x_1, \mu_1), (x_2, \mu_2), \ldots, (x_I, \mu_I)$ fall on a straight line.

• A linear trend (when it exists) is a nice way to explain differences between the mean responses μ_i in terms of the explanatory variable x_i.

• To test for a linear trend, we wish to test hypotheses

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_I \quad vs. \quad H_a : \mu_i = \beta_0 + \beta_1 x_i$, some β_0; some $\beta_1 \neq 0$

which, for $\gamma = C_1\mu_1 + C_2\mu_2 + \ldots + C_I\mu_I$, is **equivalent** to testing

$$H_0 : \gamma = 0 \quad vs. \quad H_a : \gamma \neq 0$$

using **special coefficients** $C_i = x_i - \bar{x}$ where $\bar{x} = (x_1 + x_2 + \cdots + x_I)/I$ is the sample mean of the I levels of the explanatory variable x_i

Sometimes instead of writing "C_i" for the i*th* group with level x_i, we write "C_x" to describe the group by the x value (e.g., $x = 10, 20, 30, 40$)