Comparing Two Means

During the first week of class you (briefly) saw hypothetical data from an experiment designed to compare the effect of two diets (new vs. standard) on lean percentage of hogs. The 50 hogs randomly assigned to receive the new diet had an average lean percentage of 52.3. The standard deviation of their 50 lean percentages was 2.1. The sample average and standard deviation in the control group of 50 hogs were 50.8 and 1.8, respectively. Is there significant evidence that the new diet increases lean percentage?
Suppose the following conditions hold.

- Population 1 has mean μ_1 and standard deviation σ_1.
- Population 2 has mean μ_2 and standard deviation σ_2.
- σ_1 is approximately equal to σ_2.
- A simple random sample of size n_1 from population 1 yields sample average and standard deviation $\bar{Y}_1$ and s_1, respectively.
- A simple random sample of size n_2 from population 2 (independent of the sample from population 1) yields sample average and standard deviation $\bar{Y}_2$ and s_2, respectively.
- The sample sizes n_1 and n_2 are large AND/OR populations 1 and 2 are roughly normal.

Then....

- The standard error of $\bar{Y}_2 - \bar{Y}_1$ is given by

$$\text{SE}(\bar{Y}_2 - \bar{Y}_1) = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad \text{where} \quad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}.$$

- The ratio

$$t = \frac{(\bar{Y}_2 - \bar{Y}_1) - (\mu_2 - \mu_1)}{\text{SE}(\bar{Y}_2 - \bar{Y}_1)}$$

  has an approximate t-distribution with $n_1 + n_2 - 2$ degrees of freedom.

- A $100(1-\alpha)$% confidence interval for $\mu_2 - \mu_1$ is given by

$$\bar{Y}_2 - \bar{Y}_1 \pm t_{n_1+n_2-2}^{(1-\alpha/2)}\,\text{SE}(\bar{Y}_2 - \bar{Y}_1),$$

  where $t_{n_1+n_2-1}^{(1-\alpha/2)}$ is the t-ratio larger than $100(1-\alpha/2)$% of t-ratios from a t-distribution with $n_1 + n_2 - 2$ degrees of freedom.

Back to the hypothetical experiment comparing diets.

1. Write down appropriate null and alternative hypotheses.

2. Compute a test statistic.

3. Find the p-value of this test.

4. State a conclusion.

5. Find a 95% confidence interval for the difference between the mean lean percentage of hogs fed the new diet and the mean lean percentage of hogs fed the standard diet.