STAT 401:
Some Practice Problems on CIs & Testing

1. A good estimate of the population mean μ is the sample mean $\bar{Y}$. **T F** **(T=True F=False)**

2. A 90% confidence interval for a population parameter means that if a large number of confidence intervals were constructed from repeated samples, then on average, 90% of these intervals would contain the true parameter. **T F**

3. A statement contradicting the claim in the null hypothesis about a population parameter is classified as the alternative hypothesis. **T F**

4. The null hypothesis is considered to be false until shown otherwise. **T F**

5. In the p-value approach to hypothesis testing, if the p-value is less than 0.01, we have weak evidence in favor of the alterative hypothesis. **T F**

6. The p-value of a hypothesis test can be computed without the value of the test statistic. **T F**

7. In hypothesis testing, if there is evidence against the null hypothesis, then there is evidence also against the alternative hypothesis. **T F**

8. A single estimate for the difference of two population means $\mu_2 - \mu_1$ is given by $s_2 - s_1$. **T F**

9. The distribution of the difference between two sample means is approximately normal with standard deviation $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$, where n_1 and n_2 are the respective sample sizes from populations 1 and 2 and σ_1 and σ_2 are the respective population standard deviations, if the sample sizes are both large. **T F**

10. The t-distribution is used in the construction of confidence intervals for the population mean when the population standard deviation is unknown. **T F**

11. When the data are obtained from matching or pairing, one can use two-sample t-test for making inferences with the data. **T F**

12. If we are constructing a 98% confidence interval for the population mean, the confidence level will be
(a) 2%
(b) 2.29
(c) 98%
(d) 2.39

13. In constructing a confidence interval for the population mean μ when σ is unknown, if the level of confidence is changed from 98% to 90%, the standard error of the sample mean will
(a) be equal to 90% of the original standard deviation of the sample mean.
(b) increase.
(c) decrease.
(d) remain the same.

14. Suppose the heights of the population of basketball players at a certain college are normally distributed. If a sample of heights of size 16 is randomly selected from this population with a mean of 6.2 ft and standard deviation of 2ft, the 90% confidence interval for the mean height of these basketball players is
(a) 4.555 to 7.845 ft.
(b) 5.324 to 7.077 ft.
(c) 4.447 to 7.953 ft.
(d) 5.135 to 7.266 ft.

15. A 99% confidence interval is to be constructed for a population mean from a random sample of size 22. If the population standard deviation is unknown, the table value $t_{n-1}^{(1-\alpha/2)}$ to be used in the computation is
(a) 2.576.
(b) 2.330.
(c) 2.831.
(d) 2.580.

16. New software is being integrated into the teaching of a course with the hope that it will help to improve the overall average score for this course. The historical average score for this course is 72. If a statistical test is done for this situation, the alternative hypothesis will be
(a) $H_a : \mu \neq 72$.
(b) $H_a : \mu < 72$.
(c) $H_a : \mu = 72$.
(d) $H_a : \mu > 72$.

17. When the p-value is used in testing a hypothesis, we will not reject the null hypothesis when
(a) p-value < 0.1.
(b) p-value > 0.1.
(c) p-value $= 0.01$.
(d) p-value < 0.01.

18. A real estate agent claims that the mean price for homes in a certain subdivision is \$150000. You believe that the mean price is lower. If you plan to test his claim by taking a random sample of the prices of the homes in the subdivision, the formulated set of hypotheses will be
(a) $H_0 : \mu = 150000$ vs. $H_a : \mu > 150000$.
(b) $H_0 : \mu \leq 150000$ vs. $H_a : \mu > 150000$.
(c) $H_0 : \mu = 150000$ vs. $H_a : \mu < 150000$.

(d) $H_0 : \mu = 150000$ vs. $H_a : \mu \neq 150000$.

19. For the following information

$$n = 16, \quad \mu_0 = 15, \quad \bar{Y} = 16, \quad s = 4.$$

If you are performing a right-tailed test for the population mean (i.e., $H_a : \mu > \mu_0$), then
(a) p-value is between 0.80 and 0.85.
(b) p-value < 0.05.
(c) p-value is between 0.15 and 0.2.
(d) p-value $= 1$.

20. It was reported that a certain population had mean of 27. To test this claim, you selected a random sample of size 100. The computed sample mean and sample standard deviation were 25 and 7, respectively. The appropriate set of hypotheses for this test is
(a) $H_0 : \mu = 27$ vs. $H_a : \mu > 27$.
(b) $H_0 : \mu = 27$ vs. $H_a : \mu \neq 27$.
(c) $H_0 : \mu = 25$ vs. $H_a : \mu < 25$.
(d) $H_0 : \mu = 25$ vs. $H_a : \mu \neq 25$.

21. It was reported that a certain population had mean of 27. To test this claim, you selected a random sample of size 100. The computed sample mean and sample standard deviation were 25 and 7, respectively. The computed test statistic for the appropriate set of hypotheses is
(a) -4.0816.
(b) -0.4082.
(c) -28.5714.
(d) -2.8571.

22. It was reported that a certain population had mean of 27. To test this claim, you selected a random sample of size 100. The computed sample mean and sample standard deviation were 25 and 7, respectively. The p-value for the appropriate set of hypotheses satisfies
(a) p-value $= 0.0021$.
(b) p-value $= 0.9979$.
(c) $0.0025 <$ p-value < 0.005.
(d) $0.005 <$ p-value < 0.01.

23. It was reported that a certain population had mean of 27. To test this claim, you selected a random sample of size 100. The computed sample mean and sample standard deviation were 25 and 7, respectively. Based on the p-value, you can claim that the mean of the population is
(a) not equal to 25.
(b) equal to 25.
(c) not equal to 27.
(d) equal to 27.

24. Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines

is given in the table.

	Machine 2	Machine 1
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	16	12

A single estimate for the difference between the two population means $\mu_2 - \mu_1$ is
(a) 17.
(b) 3.
(c) 4.
(d) -4.

25. Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines is given in the table.

	Machine 2	Machine 1
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	36	22

Assuming the two population standard deviations are equal $\sigma_1 = \sigma_2$, the estimate of the standard deviation (standard error) for the distribution of differences of sample means $\bar{Y}_2 - \bar{Y}_1$ is
(a) 0.9285.
(b) 0.1931.
(c) 0.3850.
(d) 0.3217.

26. *Note the column labels here.* Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines is given in the table.

	Machine 1	Machine 2
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	36	22

If you are to conduct a test to determine whether the mean amount dispensed by machine 1 is significantly more than the mean amount dispensed by machine 2, then the appropriate set of hypotheses is
(a) $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_2 > \mu_1$.
(b) $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_2 < \mu_1$.
(c) $H_0 : \mu_1 = \mu_2$ vs. $H_a : \mu_2 \neq \mu_1$.
(d) $H_0 : \mu_1 \neq \mu_2$ vs. $H_a : \mu_2 = \mu_1$.

27. *Note the column labels here.* Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines is given in the table.

	Machine 1	Machine 2
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	36	22

If you are to conduct a test to determine whether the mean amount dispensed by machine 1 is significantly more than the mean amount dispensed by machine 2, then the computed test statistic for the test is
(a) 3.2310.
(b) -3.2310.
(c) 4.8348.
(d) 2.1988.

28. *Note the column labels here.* Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines is given in the table.

	Machine 1	Machine 2
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	36	22

If you are to conduct a test to determine whether the mean amount dispensed by machine 1 is significantly more than the mean amount dispensed by machine 2, then the correct decision is
(a) do not reject the null hypothesis.
(b) reject the null hypothesis.
(c) reject the alternative hypothesis.
(d) do not reject the alternative hypothesis.

29. Two machines are used to fill 50-lb bags of dog food. Sample information for these two machines is given in the table.

	Machine 1	Machine 2
Sample Size	81	61
Sample Mean (pounds)	51	48
Sample Variance	36	22

A 90% confidence interval for the difference of the two population means $\mu_2 - \mu_1$ is
(a) -3 ± 0.3850.
(b) -3 ± 1.4760.
(c) -3 ± 1.0207.
(d) -3 ± 1.5413.

30. The matched-pair t-test is appropriate
(a) when the samples are independent.
(b) when there are two samples.

(c) when there are two measurements on each individual.
(d) in none of the above cases.

31. A group of foreign students who would like to study in the United States registered for a special TOEFL (Test of English as a Foreign Language) preparatory course offered in their home country. They took a sample examination on the first day of class and then retook it at the end of the course. The results for six of the students are given below.

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490

Such sample data would be considered
(a) independent data.
(b) dependent data.
(c) not large enough data.
(d) none of the above.

32. A group of foreign students who would like to study in the United States registered for a special TOEFL (Test of English as a Foreign Language) preparatory course offered in their home country. They took a sample examination on the first day of class and then retook it at the end of the course. The results for six of the students are given below.

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490

Let μ represent the mean of the population of differences (after score - before score). If you want to determine whether the course helped to improve the students' scores, the appropriate set of hypotheses will be
(a) $H_0 : \mu > 0$ vs. $H_a : \mu < 0$.
(b) $H_0 : \mu = 0$ vs. $H_a : \mu > 0$.
(c) $H_0 : \mu = 0$ vs. $H_a : \mu < 0$.
(d) $H_0 : \mu = 0$ vs. $H_a : \mu \neq 0$.

33. A group of foreign students who would like to study in the United States registered for a special TOEFL (Test of English as a Foreign Language) preparatory course offered in their home country. They took a sample examination on the first day of class and then retook it at the end of the course. The results for six of the students are given below.

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490

If you want to determine whether the course helped to improve the students' scores, the computed test statistic for the appropriate test is

(a) $t = -2.38$.
(b) $t = 0.02$.
(c) $t = -0.02$.
(d) $t = 2.38$.

34. A group of foreign students who would like to study in the United States registered for a special TOEFL (Test of English as a Foreign Language) preparatory course offered in their home country. They took a sample examination on the first day of class and then retook it at the end of the course. The results for six of the students are given below.

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490

If you want to determine whether the course helped to improve the students' scores, the computed p-value for the appropriate test satisfies
(a) p-value > 0.05.
(b) $0.025 <$ p-value < 0.05.
(c) p-value < 0.025.
(d) $0.95 <$ p-value < 0.975.

35. A group of foreign students who would like to study in the United States registered for a special TOEFL (Test of English as a Foreign Language) preparatory course offered in their home country. They took a sample examination on the first day of class and then retook it at the end of the course. The results for six of the students are given below.

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490

If you want to determine whether the course helped to improve the students' scores, then the evidence in favor of this based on p-value could be classified as
(a) convincing.
(b) moderate.
(c) suggestive but inconclusive.
(d) weak.

ANSWER KEY

1. T

2. T

3. T

4. F (considered true until shown otherwise)

5. F

6. F

7. F

8. F (use $\bar{Y}_2 - \bar{Y}_1$)

9. T

10. T

11. F

12. (c)

13. (d), since $\text{SE}(\bar{Y}) = \frac{s}{\sqrt{n}}$ stays the same

14. (b), 90% CI is $\bar{Y} \pm t_{n-1}^{(1-\alpha/2)} \frac{s}{\sqrt{n}}$ since sample standard deviation $s = 2$, $t_{15}^{(0.95)} = 1.753$ ($\alpha = 0.1$), sample mean $\bar{Y} = 6.2$, $n = 16$

15. (c), $t_{21}^{(0.995)} = 2.831$ with $df = n - 1 = 21$ and $\alpha = 0.01$

16. (d), want to show new average score is greater than old average 72

17. (b)

18. (c)

19. (c), the t-ratio is $t = \frac{\bar{Y}-\mu_0}{s/\sqrt{n}} = 1$ and the p-value is % of t-ratios larger than $t = 1$ (i.e., area under t-curve to right of $t = 1$) for t-distribution with $df = n - 1 = 15$. Find that 1 is between 0.866 and 1.074 in t-table ($df = 15$) so p-value is between 1-0.8 = .2 and 1-0.85 = 0.15.

20. (b)

21. (d), the test statistic is $t = \frac{\bar{Y}-\mu_0}{\frac{s}{\sqrt{n}}}$ = -2.8571; use $\bar{Y} = 25$, $\mu_0 = 27$, $s = 7$, $n = 100$

22. (d), test statistic $t = -2.8571$ so $|t| = 2.8571$ and p-value = 2× % of $t-$ratios larger than 2.8571 = 2× area under $t-$curve to the right of 2.8571 for a t-distribution with $df = n - 1 = 100 - 1 = 99$. Note that $df = 99$ is not in Table A.2, so use the next smallest degrees of freedom $df = 90$ in the following. Find 2.8571 is between 2.632 and 2.878 in table so area right of 2.8751 is between (1-0.995)=0.005 and (1-0.9975)=0.0025. So, p-value is between $2 \times 0.005 = 0.01$ and $2 \times 0.0025 = 0.005$.

23. (c), reject $H_0 : \mu = 27$ in favor of $H_a : \mu \neq 27$ since p-value is small

24. (b), use difference in sample means $\bar{Y}_2 - \bar{Y}_1 = -3$ to estimate $\mu_2 - \mu_1$

25. (a), the standard deviation of the sampling distribution of $\bar{Y}_2 - \bar{Y}_1$ is $\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ based on the population two standard deviations $\sigma_1 = \sigma_2 = \sigma$ which is estimated by the standard error SE($\bar{Y}_2 - \bar{Y}_1$) $= s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = \sqrt{30}\sqrt{\frac{1}{61} + \frac{1}{81}} = 0.9285$, involving the the pooled sample standard deviation $s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(60)22+(80)36}{61+81-2}} = \sqrt{30} = 5.477$ which is based on sample variances $s_2^2 = 36$, $s_1^2 = 22$ and sample sizes $n_2 = 81$, $n_1 = 61$

26. (b)

27. (b), the test statistic is $t = \dfrac{\bar{Y}_2 - \bar{Y}_1 - (\mu_2 - \mu_1)}{\mathrm{SE}(\bar{Y}_2 - \bar{Y}_1)} = \dfrac{48 - 51 - (0)}{0.9285} = -3.2310$ where $\bar{Y}_2 = 48$, $\bar{Y}_1 = 51$, SE($\bar{Y}_2 - \bar{Y}_1$) = 0.9285 (from above) and $\mu_2 - \mu_1 = 0$ is the assumed value under the null hypothesis H_0 of no difference in population means.

28. (b), p-value = % of t-ratios smaller than our test statistic -3.2310 = % of t-ratios larger than 3.2310 (since the t-distribution is symmetric around 0, the area in the tail to the left of -3.2310 is the same as the area in the tail to the right of 3.2310) for a t-distribution with degrees of freedom $df = n_1 + n_2 - 2 = 61 + 81 - 2 = 140$. Note $df = 140$ is not in Table A.2 so use the next smallest $df = 100$ in the following. Find 3.2310 is between 3.174 and 3.390 so p-value (i.e., area under t-curve to the right of 3.2310) is between area right of 3.174 (which is 1 - 0.999=0.001) and the area right of 3.310 (which is 1 - 0.9995 = 0.0005). Hence the p-value is very small (between 0.0005 and 0.001) so you would reject $H_0 : \mu_1 = \mu_2$ that the two population means are equal.

29. (d), 90% CI is $\bar{Y}_2 - \bar{Y}_1 \pm t^{(1-\alpha/2)}_{n_1+n_2-2}\mathrm{SE}(\bar{Y}_2 - \bar{Y}_1)$, where $\bar{Y}_1 = 51$, $\bar{Y}_2 = 48$, $n_1 = 61$ $n_2 = 81$, $\mathrm{SE}(\bar{Y}_2 - \bar{Y}_1) = 0.9285$, and $t^{(1-\alpha/2)}_{n_1+n_2-2} = t^{(0.95)}_{140}$ (since $\alpha = 0.1$). Note $df = 140$ is not in Table A.2 so use the next smallest $df = 100$, that is, use $t^{(0.95)}_{100} = 1.660$. So 90% CI is given by $48 - 51 \pm 1.660(0.9285)$.

30. (c)

31. (b), two measurements on each individual, the measurements are dependent

32. (b), want to show the 2nd score is higher than the 1st score on average, i.e. positive differences (after score - before score)

33. (d), the differences are computed below for "after score - before score"

Student	1	2	3	4	5	6
Before	325	495	525	480	525	480
After	375	520	510	515	550	490
Differences	50	25	-15	35	25	10

sample mean of differences $\bar{Y} = 130/6 = 21.667$, sample variance of differences

$$
\begin{aligned}
s^2 &= \frac{1}{n-1}\sum(y_i - \bar{y})^2 \\
&= \frac{(50-\bar{Y})^2 + (25-\bar{Y})^2 + (-15-\bar{Y})^2 + (35-\bar{Y})^2 + (25-\bar{y})^2 + (10-\bar{Y})^2}{6-1} \\
&= 496.667
\end{aligned}
$$

and the sample standard deviation is $s = \sqrt{s^2} = 22.286$. Then the test statistic is $t = \frac{\bar{Y}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{21.667-0}{\frac{22.286}{\sqrt{6}}} = 2.38$, using $\bar{Y} = 21.667$, $\mu_0 = 0$ the value of the mean difference μ under the null hypothesis ('no difference'), $s = 22.286$, $n = 6$

34. (b), p-value = % of t-ratios larger than $t = 2.38$ for a t-distribution with degrees of freedom $df = n - 1 = 5$. Find 2.38 is between 2.015 and 2.571 so p-value (i.e., area under t-curve to the right of 2.38) is between area right of 2.015 (which is 1 - 0.95=0.05) and the area right of 2.571 (which is 1 - 0.975 = 0.025). Hence the p-value is between 0.025 and 0.05.

35. (b), because p-value is between 0.025 and 0.05.