STAT401 Section F **Exam 2 Formulae** **FALL 2007**

$\bar{Y} = \frac{Y_1+Y_2+\cdots+Y_n}{n} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

$\text{SD}(\bar{Y}) = \sigma/\sqrt{n} \qquad \text{SE}(\bar{Y}) = s/\sqrt{n} \qquad t = \frac{\bar{Y}-\mu}{s/\sqrt{n}} \qquad$ d.f.$= n-1 \qquad \bar{Y} \pm t_{n-1}^{(1-\alpha/2)}\frac{s}{\sqrt{n}}$

$\text{SD}(\bar{Y}_2 - \bar{Y}_1) = \sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \qquad \text{SE}(\bar{Y}_2 - \bar{Y}_1) = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \qquad s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$

$t = \frac{(\bar{Y}_2-\bar{Y}_1)-(\mu_2-\mu_1)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \qquad$ d.f.$= n_1+n_2-2 \qquad (\bar{Y}_2 - \bar{Y}_1) \pm t_{n_1+n_2-2}^{(1-\alpha/2)} s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

Source	D.F.	Sum of Squares	Mean Squares	F
Between	$I-1$	$SSB = \sum_{i=1}^{I} n_i(\bar{Y}_i - \bar{Y})^2$	$MSB = \frac{\sum_{i=1}^{I} n_i(\bar{Y}_i-\bar{Y})^2}{I-1}$	$\frac{MSB}{MSW}$
Within	$n-I$	$SSW = \sum_{i=1}^{I}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2$	$MSW = \frac{\sum_{i=1}^{I}\sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_i)^2}{n-I}$	
Total	$n-1$	$SSTO = \sum_{i=1}^{I}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y})^2$		

$SSTO = (n-1)s_Y^2 \qquad s_p = \sqrt{\frac{\sum_{i=1}^{I}(n_i-1)s_i^2}{n-I}} = \sqrt{MSW}$

$$F = \frac{[\text{RSS(red.)} - \text{RSS(full)}]/[\text{df}_{\text{RSS(red.)}} - \text{df}_{\text{RSS(full)}}]}{\text{RSS(full)}/\text{df}_{\text{RSS(full)}}}$$

$\gamma = C_1\mu_1 + C_2\mu_2 + \cdots + C_I\mu_I \qquad g = C_1\bar{Y}_1 + C_2\bar{Y}_2 + \cdots + C_I\bar{Y}_I \qquad \text{Mean}(g) = \gamma$

$\text{SE}(g) = s_p\sqrt{\frac{C_1^2}{n_1}+\frac{C_2^2}{n_2}+\cdots+\frac{C_I^2}{n_I}} \qquad t = \frac{g-\gamma}{\text{SE}(g)} \qquad$ d.f.$= n-I \qquad g \pm t_{n-I}^{(1-\alpha/2)}\text{SE}(g)$

Bonferroni-correction : use $t_{df}^{(1-\alpha/(2m))}$ or multiply each $p-value$ by m, truncating at 1.

$$r = r_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})/(n-1)}{s_X s_Y}$$

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X \qquad \hat{\beta}_1 = \frac{r \cdot S_Y}{S_X} \qquad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \qquad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-2}} = s_Y \sqrt{(1 - r^2)\frac{n-1}{n-2}}$$

$$\text{Mean}(\hat{\beta}_1) = \beta_1 \qquad \text{SE}(\hat{\beta}_1) = \hat{\sigma}\sqrt{\frac{1}{(n-1)s_X^2}} \qquad t = \frac{\hat{\beta}_1 - \beta_1}{SE(\hat{\beta}_1)} \qquad d.f. = n-2$$

$$\text{Mean}(\hat{\beta}_0) = \beta_0 \qquad \text{SE}(\hat{\beta}_0) = \hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{(n-1)s_X^2}} \qquad t = \frac{\hat{\beta}_0 - \beta_0}{SE(\hat{\beta}_0)} \qquad d.f. = n-2$$

$$\text{SE}(\hat{\text{Y}} \mid \text{X}_0) = \hat{\sigma}\sqrt{\frac{1}{\text{n}} + \frac{(\text{X}_0 - \bar{\text{X}})^2}{(\text{n}-1)\text{s}_\text{X}^2}}$$

Source	D.F.	Sum of Squares	Mean Square	F	P-value
Regression	1	$\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$	$\frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{1}$	$\frac{MSR}{MSE}$	$P(T^2 \geq \frac{MSR}{MSE})$ $T^2 \sim F(1, n-2)$
Error	$n-2$	$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$	$\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-2}$		
Total	$n-1$	$\sum_{i=1}^{n}(Y_i - \bar{Y})^2$			

$$\hat{\sigma} = \sqrt{MSE}$$

$$R^2 = 1 - \frac{SSE}{SSTO} = \frac{SSR}{SSTO}$$

$$\hat{Y} \pm t_{n-2}^{(1-\alpha/2)}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_X^2}} \qquad \hat{Y} \pm t_{n-2}^{(1-\alpha/2)}\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_X^2}}$$

$$\hat{\beta}_0 + \hat{\beta}_1 X \pm \sqrt{2F_{2,n-2}^{(0.95)}}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_X^2}}.$$
