November 12, 2007 NAME: key

Stat 401 Section F - Fall 2007 - Exam 2

• Please write your answers in the enclosed spaces. If you need more room, continue on the back of the page. If that is not enough, ask for extra paper
•Write your name on top of each page.
•Tables and a formula sheet are attached to the exam.
•Each question is worth 4 points.
•Partial credit will be given ONLY for clearly and neatly written answers. No partial credit for True/False or multiple choice questions.
•You must include all the formulas you used and the values you chose to use in your calculations.
• Follow the *Honor Code*: neither give nor receive unauthorized help.

I. *ANSWER THIS ONE LAST*: This exam is worth 100 points. What is your best guess of what your score will be?

II. **TRUE or FALSE:** Identify which statements are valid. **(circle T or F for each.)**

(1) If your computed correlation coefficient $r = +1.2$, then you have better than a perfect positive correlation. T F $-1 \leq r \leq 1$

(2) If the correlation between two variables is zero, then the variables do not have a strong relationship of any kind. T F no linear relationship.

(3) If the least squares regression line is $\hat{Y} = 3 - 5X$ and $r^2 = 0.81$, then $r = 0.9$. T F

$r = -0.9$

III. You have just started to analyze some data. You got your output from SAS, but some of the lines are smudged and unreadable. Here is as much of the ANOVA table as you can read.

Source	d.f.	Sums of squares	Mean square	F ratio
Model	1	0.0042	0.0042	1.08
Error	6	0.0232	0.0039	
Corr. Total	7	0.0274		

(1) Please complete the blanks in the table. How many observations were used in this study?

$n=8$

(2) Could this table be describing a simple linear regression? Why or why not?

yes, since "model" line has d.f. = 1

(3) Suppose that this ANOVA table is to compare various group means. In this case, carefully state what specific *null* and *alternative* hypotheses are being tested.

if this is one-way ANOVA, then $I=2$

$H_0: \mu_1 = \mu_2$ vs. $H_a: \mu_1 \neq \mu_2$

(4) State an appropriate conclusion from this analysis.

p-value > 0.1

no evidence of difference between the 2 means.

IV. In a study of crop losses due to air pollution, plots of Blue Lake snap beans were grown in open-top field chambers, which were fumigated with various concentrations of sulfur dioxide (as shown in the table below). After a month of fumigation, the plants were harvested and the total yield of bean pods was recorded for each chamber. Total yield (in kilograms) obtained for each of the plots is given in the table:

	Group			
	I	II	III	IV
Sulfur Dioxide Concentration(ppm)	0	0.06	0.12	0.30
	2.57	0.83	1.03	0.40
	1.75	1.91	1.39	0.86
	2.04	1.63	1.56	1.22
		1.71		0.29
Sample Mean	2.12	1.52	1.33	0.69
Coefficients	$-1/3$	$-1/3$	$-1/3$	1

The within-groups sum of squares equals 1.72. Variance of all the yield values equals 0.411.

$n = 14$; $I = 4$ (groups)
$SSW = 1.72$; $s_y^2 = 0.411$

(1) Give the pooled estimate of spread s_p.

$$s_p = \sqrt{MSW} = \sqrt{\frac{SSW}{14-4}} = \sqrt{\frac{1.72}{10}} = 0.415$$

(2) **One of the goals of this study was to estimate the effect of high levels of pollutants in the air, compared to lower levels.** In the table above (bottom line), write the coefficients for the linear combination to estimate the difference between the highest sulfur dioxide treatment and the average of the three lower sulfur dioxide treatments.

Many possibilities here, $(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1)$ or $(-1, -1, -1, 3)$, or $(1/3, 1/3, 1/3, -1)$

(3) The researchers were **also interested in testing for a linear trend between sulfur dioxide level and the mean yield of bean pods.**

Estimate the contrast that tests for a linear trend in the mean yields of bean pods and its standard error. Complete the test by stating: the hypotheses in terms of a parameter of interest, the degrees of freedom, a p-value, and a brief conclusion.

$\gamma = c_1\mu_1 + c_2\mu_2 + c_3\mu_3 + c_4\mu_4$ where $c_i = x_i - \bar{x}$

$\bar{x} = \frac{0 + 0.06 + 0.12 + 0.30}{4} = 0.12 \Rightarrow c_1 = -0.12,\ c_2 = -0.06,$
$c_3 = 0,\ c_4 = 0.18$

$\Rightarrow g = c_1\bar{y}_1 + c_2\bar{y}_2 + c_3\bar{y}_3 + c_4\bar{y}_4 = (-0.12)2.12 + (-0.06)1.52 + (0)1.33 + (0.18)0.69 = 0.2214$

$SE(g) = s_p\sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \frac{c_3^2}{n_3} + \frac{c_4^2}{n_4}} = 0.04875 \Rightarrow \begin{cases} t = \frac{g - 0}{SE(g)} = -4.54 \\ d.f. = 10 \end{cases}$

$\begin{cases} H_0: \gamma = 0 \\ H_a: \gamma \neq 0 \end{cases}$ and 2-sided p-value betw. 0.001 and 0.002 $\Rightarrow$ strong evidence against H_0 $\Rightarrow$ slope is NOT zero

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(4) Conduct a lack-of-fit test for this problem. You may find it useful to use the following **R** output:

```
Call:
lm(formula = beans ~ so2)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.9190     0.1616  11.873 5.44e-08 ***
so2          -4.2643     0.9358  -4.557 0.000658 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4038 on 12 degrees of freedom
Multiple R-Squared: 0.6338,Adjusted R-squared: 0.6032
F-statistic: 20.77 on 1 and 12 DF,  p-value: 0.0006583
```

$RSS_{full} = RSS_{res\ ANOVA} = RSS_{Within} = 1.72$; $d.f. = 14-4 = 10$

$RSS_{red} = RSS_{res\ Regr} = RSS_{error} = (0.4038)^2 \times 12 = 1.96$; $d.f. = 12$

alternatively, recall that for regression $R^2 = 1 - \frac{SSE}{SSTO} \Rightarrow SSE = SSTO(1-R^2)$ $\Big\} \Rightarrow SSE = 1.96$

knowing that $R^2 = 0.6338$ and $SSTO = (n-1)S_y^2 = 13 \times 0.411 = 5.343$

$$\Rightarrow F = \frac{(1.96-1.72)/(12-10)}{1.72/10} = \frac{0.12}{.172} = 0.7 \Rightarrow \text{retain } H_0 \Rightarrow \text{regr. is OK.}$$

(5) **Another goal of the study was to test for differences among the means of the** ***three*** **groups which involved a non-zero presence of sulfur dioxide.** This question can be answered by comparing two models.

i. Using words or symbols (μ_i's), describe the two models:

full model: all μs are different $\rightarrow$ 4 parameters

reduced model: $\mu_2 = \mu_3 = \mu_4$ and μ_1 different $\Rightarrow$ 2 parameters.

ii. For the model comparison described above, list estimates of the group means under both models and the degrees of freedom for the residual sums of squares (RSS):

full model

Group	I	II	III	IV
Estimate of Mean	2.12	1.52	1.33	0.69

df: $14-4=10$

reduced model

Group	I	II	III	IV
Estimate of Mean	2.12	1.17	1.17	1.17

df: $14-2=12$

1.17 = average of obs. in gr. 2,3,4

(6) To perform comparisons among all *four* groups, one possibility is to construct confidence intervals for each pair of differences $\mu_i - \mu_j$ between any two group means (6 in total). If you were to set all the individual confidence levels at 76%, would the familywise confidence level (e.g., the probability that, simultaneously, all pairwise confidence intervals contain their parameters) be smaller, larger or about 76%?

smaller

(7) Suppose you use the Bonferroni method to construct simultaneous/familywise 76% intervals for part (6). Using this Bonferroni adjustment, what would be the margin of error (ME) for the confidence interval "$\bar{Y}_1 - \bar{Y}_2 \pm ME$" estimating $\mu_1 - \mu_2$?

$m=6$; $\alpha = 1-0.76 = 0.24 \Rightarrow 1-\frac{\alpha}{2 \cdot m} = 1 - \frac{0.24}{2 \cdot 6} = 0.98$

$d.f. = n - I = 10 \Rightarrow t_{10}^{0.98} = 2.359$

For $\bar{y}_1 - \bar{y}_2$ the ME is $t_{10}^{.98} \times SE(\bar{y}_1 - \bar{y}_2)$ or $t_{10}^{.98} \times s_p \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

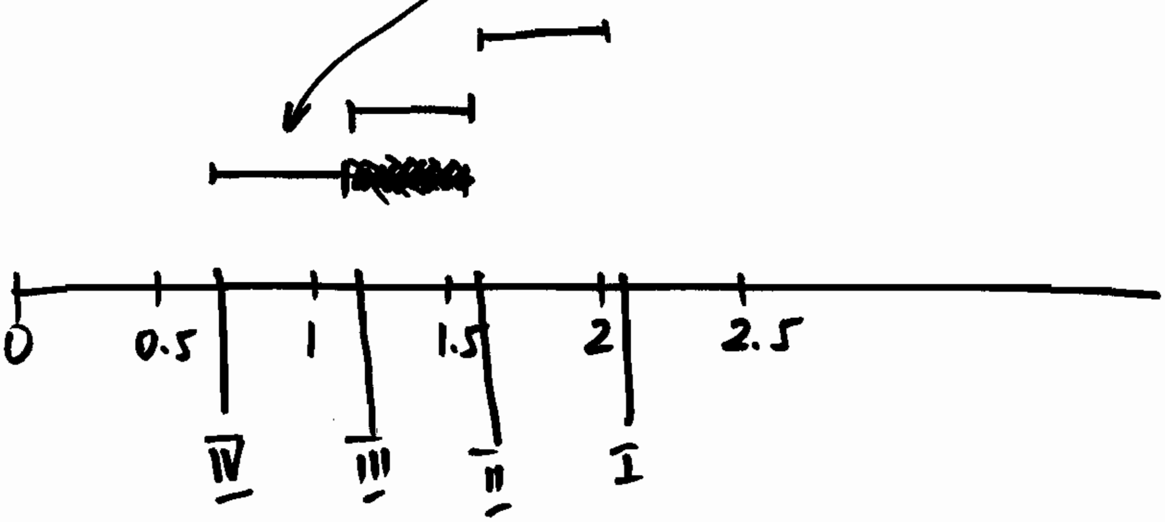
$\Rightarrow 2.359 \times 0.415 \times \sqrt{\frac{1}{3} + \frac{1}{4}} = 0.748$

(8) Suppose the minimum significant difference associated with the Tukey method for all pairwise comparisons among the four means turned out to be 0.7 (it's not really). Use this information to construct a line plot that illustrates which treatment means are significantly different from each other.

HSD = 0.7 ⇒ signif. differences are:

(I, III)
(I, IV)
(II, IV)

groups:	I	II	III	IV
I		0.6	0.79	1.43
II			0.19	0.83
III				0.64



V. Behavioral scientists are interested in patterns of interactions between wives and husbands. However, they suspect that people are poor reporters of their own behavior. To demonstrate this, they recruited a sample of 36 married couples. These couples were first videotaped while discussing a family problem. After the problem discussion session ended, each husband was asked to complete a questionnaire. The questionnaire contained a series of 4 questions that asked husbands to report their level of hostility toward their wives during the session. Examples of the questions are, "During the problem discussion session, how often did you argue with your spouse?", and "How often did you get angry at something she said?" Responses were on a seven point scale from "never" (scored "1") to "always"(7). The 4 questions were summed to create an index that ranged from 4 to 27. The index was labeled "self-reported hostility".

In addition, specially trained videotape coders watched the videotaped discussions and rated the behaviors of husbands toward their wives (e.g., anger, positive reinforcement). Their ratings were used to create an index of "observed hostility" of the husbands toward their wives. The index ranged from 4 to 34. The videotape coders also rated each couple on their degree of supportiveness toward each other. Each of the 36 couples was rated as having low, moderate or high supportiveness.

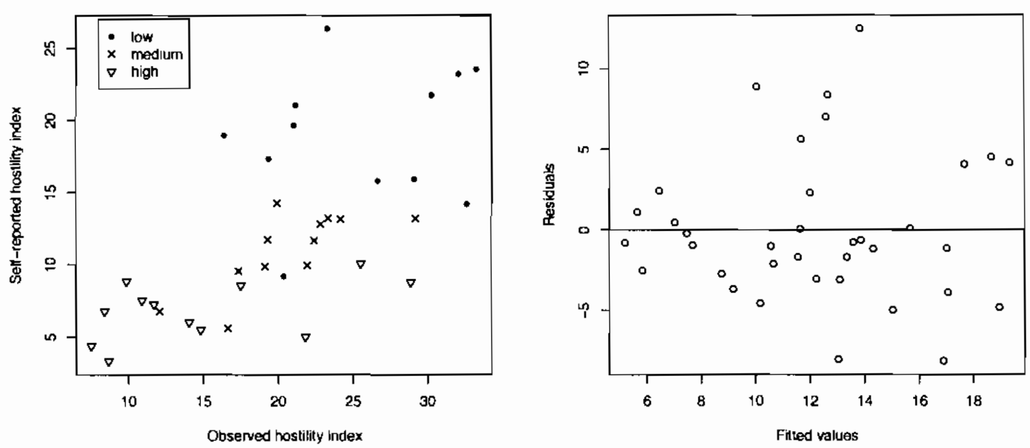


Figure 1: Scatterplot of husbands' self-reported hostility against observed hostility. Different symbols represent the level of supportiveness (low (•), medium (x), high (∇)).

The behavioral scientists want you to help them interpret their analyses. They have compiled the following descriptive statistics, including means and standard deviations:

		Self-reported hostility		Observed hostility	
Level of supportiveness	n	Mean	Std. dev	Mean	Std. dev
Low	12	19.33	5.35	24	4.79
Moderate	12	11.67	4.18	20.58	5.63
High	12	6.58	1.93	15	7.14
Total	36	12.53	6.63	19.86	6.88

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Part A: Using the data on husbands' behaviors toward their wives, the scientists regressed self-reported hostility (Y) against observed hostility (X). They obtained the following prediction equation, regression sums of squares (SSReg), and mean square error (MSE):

$$\hat{Y}_i = -1.47 + 0.705X_i \qquad SSReg = 823 \qquad MSE = 21$$

(1) The slope in the above simple linear regression line seems pretty small. Compute the proportion of the variance in Y explained by X in this model.

$R^2 = \frac{SSReg}{SSTO}$ and $SSTO = (n-1)S_y^2 = (36-1) \times 6.63^2 = 1538.491$

$R^2 = \frac{823}{1538.491} = 0.53$ or 53%

alternatively, $SSE = MSE \times (n-p) = 21 \times 34$

(2) Does these data provide evidence that the slope is significantly different from zero? (write down the formal hypothesis, provide a test statistic, degrees of freedom and $p-value$ along with your conclusions).

$F = \frac{MSReg}{MSE} = \frac{823/1}{21} = 39.6$ with $df = (1, 34) \Rightarrow p\text{-val} \simeq 0$

(look up in t-table with d.f. = 34 for $\sqrt{39.6}$)

Significant evidence that slope $\neq 0$

(3) Based on the description of the study, what would be a more appropriate alternative hypothesis for the slope? Construct a 90% confidence interval for the slope and comment on the significance of the relationship based on this interval.

$H_a: \beta_1 > 0$ Know $\hat{\beta}_1 = 0.705$

Also have from (2) that $F = 39.6 \Leftrightarrow \left[\frac{\hat{\beta}_1}{SE(\hat{\beta}_1)}\right]^2 = 39.6 \Rightarrow SE(\hat{\beta}_1) = \sqrt{\frac{.705}{39.6}} = .112$

$\Rightarrow 90\%CI \Rightarrow .705 \pm 1.697 \times 0.112 = (0.51, 0.89) \Rightarrow \beta_1 > 0$ with 95% confidence

(4) Construct a 90% confidence interval for the mean value of self-reported hostility when observed hostility is equal to 15.

$X = 15 \Rightarrow \hat{y}_{15} = 9.1$

$t_{34}^{.95} = 2.032$

$SE(\hat{y}_{15}) = \sqrt{MSE}\sqrt{\frac{1}{36} + \frac{(15-19.86)^2}{35 \times 6.88^2}} = 0.94$

$\Rightarrow (7.19, 11.01)$

(5) *Sketch* the residual plot for this model in the space provided next to the scatterplot of the data (Figure 1). Based on this, what can you conclude about the validity/appropriateness of the simple linear regression model here?

Some indication of unequal variances.

Part B: The above analysis does not take into account the possibility that the relationship between husbands' self-reported and observed hostility might be affected by couples level of supportiveness. To take this into account, the researchers fit four models to the data. The estimated regression models, regression sums of squares and mean square errors are shown below:

Model		SSReg	MSE
$M_1:$	$\hat{Y}_i = -1.47 + 0.71X_i$	823	21
$M_2:$	$\hat{Y}_i = 19.33 - 7.67S1_i - 12.75S2_i$	989	16.61
$M_3:$	$\hat{Y}_i = 9.20 + 0.42X_i - 6.22S1_i - 8.95S2_i$	1196	10.66
$M_4:$	$\hat{Y}_i = -3.81 + 0.96X_i + 7.20S1_i + 7.53S2_i - 0.56X_iS1_i - 0.77X_iS2_i$	1300	7.90

where S1 and S2 are indicator variables defined as $S1 = 1$ if level of supportiveness was moderate and 0 otherwise, $S2 = 1$ if the level of supportiveness was high and 0 otherwise.

The behavioral scientists ask you to help them interpret the analyses of these models:

(6) Start with model M_2. Within the context of this hostility study, interpret the null hypothesis $H_0 : \beta_1 = 0$ where β_1 is associated with $S1$ in this model.

It's best to write the form of the model for each group:
$\rightarrow$ high: $y = \beta_0 + \beta_2$
$\rightarrow$ Mod: $y = \beta_0 + \beta_1$
$\rightarrow$ low: $y = \beta_0$

Hence β_1 is the difference in means between "moderate" and "low" group. If $\beta_1 = 0 \Rightarrow$ no diff in means between these two groups.

(7) Next, the scientists are concerned about the interaction. Examine the results from fitting the four models for evidence of interaction effects (conduct a test and base your conclusions on the resulting $p-value$; you can use the information that the F-ratio with 2 and 30 degrees of freedom is 5.39 when α=0.01.)

Compare models M_3 and M_4 to answer this: M_3 reduced, M_4 full.

Full: $df = 36-6 = 30$; $RSS = 7.9 \times 30 = 237$

Red: $d.f. = 36-4 = 32$; $RSS = 10.66 \times 32 = 341.12$

$$\Rightarrow F = \frac{(341.12 - 237)/(32-30)}{237/30} = 6.58$$

p-value smaller than 0.01 $\Rightarrow$ interaction effects present.

(8) Part of your role is to interpret the meaning of interaction effects. Explain how the relationship between the husband's self-reported and observed hostility for couples with low levels of supportiveness differs from the relationship between self-reported and observed hostility for couples with moderate levels of supportiveness.

Again, write out Model M_4 for the 3 groups:

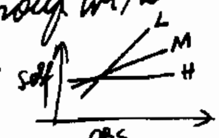
low: $y = -3.81 + 0.96X$

moderate: $y = -3.81 + 0.96X + 7.20 - 0.56X = 3.39 + 0.4X$

high: $y = -3.81 + 0.96X + 7.53 - 0.77X = 3.72 + 0.19X$

slopes are: low: 0.96, mod: 0.4, high: 0.19 — decrease in value

Largest slope for the group with lowest supportiveness.



(9) Select the most appropriate model and obtain the predicted value of husbands' self-reported hostility when supportiveness is moderate and observed hostility is 19.

Since there is evidence of interaction, choose model M_4.

$\hat{y} = 3.39 + 0.40 \times 19 = 11.01.$