Worksheet – Normal Distributions

1. For each problem also draw a picture of the normal curve and shade the area you have to compute. Let \( Z \) represent a variable following a standard normal distribution.

(a) Find the proportion that is less than \( z = 2.00 \).

area below the curve corresponding to a \( z \)-score of \( z = 2.0 \) or less is \( 0.9772 \).

(b) Find the proportion that is between \( z = 1.75 \) and \( z = 0.13 \).

\[
\text{area under the curve between } 0.13 \text{ and } 1.75 \text{ equals the area to the left of } 1.75 - \text{ the area to the left of } 0.13 = 0.9599 - 0.4483 = 0.5116
\]

\( z = 0.13 \text{ } z = 1.75 \)

(c) Find the proportion that is greater than \( z = 1.86 \).

the area to the right of 1.86 can be found by taking \( 1 - (\text{area to the left of } 1.86) = 1 - 0.9686 \) or by finding the area left of \( -1.86 \) : \( 0.0314 \)

\( z = 1.86 \)

(d) Find the \( z \)-score for the 64th percentile.

We now have an area of 0.64 given and are looking for the corresponding \( z \)-score; find the \( z \)-score value to 0.64 inside the table \( \Rightarrow z = 0.36 \)

\( z = 0.64 \)

(e) Find the \( z \)-scores that bound the middle 50% of all data.

the \( z \)-score that bounds the middle 50% correspond to the \( z \)-scores of the 25th and 75th percentile; .25 do the same as above in d) \( \Rightarrow z = -0.67 \) & \( z = 0.67 \)

(f) Find the \( z \)-score for the 24th percentile.

the same as in d) and e); we have an area of 0.24 given and need to find it's \( z \)-score .24 \( z = -0.71 \)

2. Former ISU basketball player Kelvin Cato is 83 inches tall.

(a) What is his corresponding \( z \)-score?

Still assume a mean \( \mu = 70 \) \( \sigma = 3 \) \( \Rightarrow z = \frac{83 - 70}{3} = 4.33 \)

(b) What proportion of men are taller than him?

4.33 is off the chart. The highest \( z \)-score we can find in the table is 3.49; the area above (1-area below) 3.49 is almost zero, so the area above 4.33 is even smaller.

\( \Rightarrow \) virtually nobody is taller than him.
3. Since the length of a downhill ski is related to the height of the individuals renting them, it is fair to assume that a normal distribution would describe the length of women's skis at rental outlets in Colorado. The mean of the distribution is 150 cm and the standard deviation is 10 cm.

(a) What is the proportion of women's ski lengths that are less than 130 cm?

\[ z = \frac{130 - 150}{10} = -2.00 \rightarrow \text{area to the left of } z = -2.00 \text{ is } 0.0228 \]

(b) What is the proportion of women's ski lengths that are greater than 125 cm?

\[ z = \frac{125 - 150}{10} = -2.5 \rightarrow \text{area above } z = -2.5 \text{ is } 1 - (\text{area below } -2.5) = 1 - 0.0062 = 0.9938 \]

(c) What is the proportion of women's ski lengths that are between 125 and 155 cm?

\[ z = \frac{155 - 150}{10} = 0.5 \rightarrow \text{area between } z = 0.5 \text{ is } 0.6915 - 0.0062 = 0.6853 \]

(d) Very long skis are expensive and there are not many people who rent them. What is the longest women's ski a rental shop should carry so that only 2 percent of the customers will ask to rent a longer ski?

\[ z = \frac{130 - 150}{10} = -2.00 \rightarrow \text{area to the left of } z = -2.00 = 0.0228 \]

4. The BMI for males age 20 to 74 is follows approximately a normal distribution with mean \( \mu = 27.9 \) and standard deviation \( \sigma = 7.8 \). Use the 68-95-99.7 rule to find

(a) the percentage of males with BMI less than 20.1.

20.1 is 1 \( \sigma \) below the mean, because it's the middle 68\%, we need to split the remaining 32\% equally \( \Rightarrow 16\% \)

(b) the percentages of males with BMI greater than 12.3.

12.3 corresponds to 2 \( \sigma \) below the mean \( \Rightarrow 97.5\% \) are greater

(c) the BMI values that correspond to the middle 99.7\% of the distribution.

The middle 99.7\% correspond to 3 \( \sigma \) below and 3 \( \sigma \) above the mean \( \Rightarrow 27.9 - 3 \times 7.8 = 4.5 \)

\[ 27.9 + 3 \times 7.8 = 51.3 \]

(d) the value such that 0.15\% of males have BMI's greater than the value.

51.3 corresponds to 3 \( \sigma \) above the mean, i.e. the middle 99.7\% which leave 0.1598 to the right and left of 51.3

\[ 2.5 \times 7.8 = 19.5 \]